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A STUDY OF THE EFFECT OF A
SEMI-FLEXIBLE CONNECTION
IN ARTICULATED WEDGE-BEAM FRAMING

LUTHER TAYLOR CHESNUT, 3RD
PAUL OTTO GADDIS

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A STUDY OF THE EFFECT OF A SEMI-FLEXIBLE CONNECTION IN ARTICULATED WEDGE-BEAM FRAMING

Luther Taylor Chesnut 3rd

Paul Otto Gaddis

Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Civil Engineering
Rensselaer Polytechnic Institute
Troy, New York
June 1949

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I. ACKNOWLEDGMENTS

In the autumn of 1948 the authors of this thesis visited Mr. Arsham Amirikian, Head Designing Engineer with the Department of Yards and Docks, U. S. Navy, at his office in Washington. At that time the authors expressed to him their wish to engage in thesis work which would be directly relevant to some practical problem then existing in the structural field. It was as a result of Mr. Amirikian's suggestions that the present topic was adopted. The authors are deeply grateful to Mr. Amirikian for his inspiration and for his critical evaluation of the work as it was carried out, and it is their hope that this work will in turn prove of some value to his concept of wedge-beam framing.

The authors wish to acknowledge with appreciation the assistance of Admiral Lewis B. Combs, Head of the Department of Civil Engineering at Rensselaer Polytechnic Institute, in obtaining some of the literature and equipment used in this thesis. They are likewise grateful to Professor J. Sterling Kinney for his many helpful suggestions as faculty advisor, to Professor E. J. Kilcawley for his generous loan of certain precise instrumentation devices, and to Assistant Professor A. J. Fairbanks of the Department of Aeronautical Engineering for his aid in obtaining experimental apparatus. Professor Elmo Timbey of Princeton University generously provided the authors with his personal copy of a valuable out-of-print book on model analysis.

The authors acknowledge their particular indebtedness to Mr. Clint Milliken, laboratory instructor in the Department of Metallurgical Engineering at Rensselaer Polytechnic Institute, for his excellent welding on the models and for his invaluable aid in construction of the models.

LUTHER TAYLOR CHESNUT 3rd

PAUL OTTO GADDIS

Troy, New York
June 1949

INTRODUCTION

The broad objective of this work is the determination of the effect of a semi-flexible welded connection acting in a single-span steel rigid frame.[1] More specifically, the objective of the study is a comparison of the action of an articulated frame under gravity loading with the action of a semi-articulated frame under the same loading. The articulated frame is defined as a single-span steel frame, having rigid joints at the knees, pinned connections at the base of each column, and a pin connection at the crown. The semi-articulated frame is exactly similar to the articulated frame, with the modification that the pinned connection at the crown in the latter is replaced by a semi-flexible welded strap connection in the former.

The need for an investigation of the action of the semi-flexible welded type of connection has arisen in conjunction with theoretical work now being done on articulated wedge-beam framing by Mr. Arsham Amirikian, head designing engineer with The Bureau of Yards and Docks, U. S. Navy. Mr. Amirikian has exhaustively covered the theoretical aspects of this new type of framing and will soon publish the complete theory, including analytical formulae and tables for use in the construction industry.

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1. The particular frame involved in this study will differ from the conventional concepts because of the use of wedge-beam members, as will be explained later.

A brief outline of the basic principles involved in wedge-beam framing may be found in a preliminary article.[1] Conventional design methods do not fully utilize the materials in flexural members, in that they require the choice of an entire member based on some maximum stress occurring at only one point in the member. Wedge-beam framing involves the use of tapered beams (See Fig. 2) which are designed to furnish a varying section modulus to conform roughly to certain moment diagrams, thus avoiding large portions of understressed materials.

The concept of an articulated frame composed of wedge-beam members is based on the action of a simple cantilever beam in flexure. The simple cantilever beam in flexure exhibits a unique characteristic which distinguishes it from any other member in flexure, i.e., it has a consistent pattern of bending since the moment diagram inevitably varies from a minimum at the free end to a maximum at the fixed end, no matter what system of lateral loading is applied. This postulate is illustrated in Fig. 1. Referring to Fig. 2, it is seen that the introduction of a pin at the center of the horizontal member of a frame causes simple cantilever action in the two girder members thus formed, the pinned ends being analogous to free ends. Moreover, if these girder members are wedge-beams, the taper of each girder may be made to furnish a section modulus which varies along the length of the girder in close accord with the section modulus required by flexure.

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[1] Amirikian, A. "Future Developments in Welded Steel Buildings". The Welded Journal, XVII (Aug. 1948) 593-599.

This explanation of the underlying principles of wedge-beam framing is admittedly brief and is presented in the nature of a background to the main topic, since this work does not concern itself primarily with the wedge-beam theory. Also, any discussion of the expected savings in cost and weight resulting from the application of this theory is considered outside the scope of this study. These advantages of wedge-beam framing are treated at length in Mr. Amirikian's work. However, it can be readily be seen that the economic advantages of wedge-beam framing are nullified to some extent by the necessity of providing a true hinged connection at the crown of the frame, since this type of connection is relatively expensive. If an inexpensive welded strap could be substituted for the crown hinge, without materially changing the elastic action of the frame under loading, then the economic drawback associated with the crown hinged connection could be avoided, and at the same time the complete theoretical derivations made by Mr. Amirikian for the case of the true hinged connection would hold valid. Herein lies the practical justification for the pursuit of the study of the effect of a semi-flexible welded connection.

III. GENERAL DISCUSSION

3-1. Method of Attack. The method of structural model analysis lends itself readily to an investigation of the effect of a semi-flexible connection. Model analysis may be defined briefly as an attempt to simulate field conditions in a laboratory by the use of a laboratory structure smaller than the field structure but nevertheless representative of it. It is of course necessary to interpret the behavior of the laboratory structure, hereinafter called the model, in order to determine the probable behavior of the field structure, hereinafter called the prototype.

The use of model analysis is currently in favor in the structural engineering field, but it has not always been so. The basic principles of model analysis, generally known as the laws of similitude between two systems, have been established for a relatively long period of time,[1] but their application to structural analysis has been painfully slow. A precise technique and a good understanding of the laws of similitude as they apply to the interpretation of model behavior have only been attained in the last twenty-five years. Greater faith in model analysis has led to the expenditure of large sums of money for well-equipped model laboratories by both governmental and private agencies. These laboratories, and the important results obtained therein, have caused the general acceptance of the method of model analysis.

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[1] Newton. Principia, Book II. (1687).

Engineering literature generally gives three possible reasons for the use of model analysis:

(1) Mathematical analysis of the problem concerned is virtually impossible.

(2) Mathematical analysis is possible but it is complex and tedious to a degree that justifies a short cut by model analysis.

(3) The importance of the problem is such that verification of the mathematical solution by model testing is warranted.

The investigation of the effect of a semi-flexible connection in wedge-beam framing fits under reason (1) above because of the inherent indeterminateness of the particular welded strap connection involved in this problem. However, there is an additional reason for the use of model analysis in this study. Designers in the field will be most hesitant to accept any theory differing from conventional framing methods as does wedge-beam framing unless very thorough experimentation backs up that theory. Comprehensive experiments and tests will be required to give validity and emphasis to the theoretical derivations of wedge-beam framing. It is hoped by the authors that this work will prove to be one part of the needed experimentation.

In the application of model analysis to this investigation, a prototype wedge-beam structure was first designed. Although the advantages of the new framing are more marked in multiple-

bay frames of one or more stories (and in fact the original intent of Mr. Amirikian was that the application would be to multiple-bay framing) it was felt that a single-bay frame would be sufficient for all purposes of this investigation. This frame was designed in accordance with modern methods.[1]

The next main step in attacking the problem was the computation of theoretical deflections and deflection angles in the prototype acting as an articulated frame. These quantities were first calculated by an employment of the well known method of vitual work, and were subsequently checked by formulae furnished by Mr. Amirikian which were extensions of the slope-deflection principle.

Next it was necessary to design and construct a model or models. There were two alternatives:)1) One model could be built with an interchangeable type of connection at the crown, i.e., a true hinged connection or a welded semi-flexible connection could be substituted one for the other at the crown, and experiments could be run with each type of connection acting, or (2) two identical models could be built, one with a hinged connection permanently attached at the crown and the other with a semi-flexible connection at the same location. The first alternative was not used because of the uncertainties of welding and re-welding the respective connections at the crown of the frame. The authors believed that the comparatively

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1. Griffiths, John D. Single Span Rigid Frames in Steel. (New York: Oct. 1948)

larger amount of welding required in the neighborhood of the crown by the first alternative would cause local distortions and would destroy the vital elastic similarity of the frame itself for each test. Therefore two models were designed and constructed.

Finally, the instrumentation for measuring deflections was designed, the actual experimentation was conducted, and the data from this experimentation was interpreted with regard to its bearing on the problem being investigated.

A summary of the procedure of this study is given in the following outline:

1. Design of typical prototype frame by method currently used in the industry.
2. Computation of deflections and deflection angles in prototype by theoretical analysis.
3. Employment of laws of similitude to deduce proportions of model from those of prototype.
4. Preliminary testing of steel used in models, and testing of methods of constructing model.
5. Design of model structure and its supporting framework.
6. Construction of model components and erection of model.
7. Design and construction of instrumentation for measuring deflection quantities.
8. Placement and adjustment of model in testing position.
9. Application of load conditions and measurement of resulting deflections.
10. Translating and summarising the observed data.

3-2. Design Data and Assumptions. The dimensions chosen for the prototype wedge-beam frame were a 40 ft. span and a 12 ft. height. The span length was measured from center line of column to center line of column, and the height was measured from the pin at the base of the column to the center line of the girder. See Fig. 2. The distance from the center line of one column to the crown hinge was a half-span length, or 20 ft.

A uniform load of 1000 ppf or 1 kpf was assumed to act along the upper flange of the girder member. For the purposes of this study it was permissible to consider this load as the total load acting on the frame, including dead load and live load. The dead load would consist of the weight of the frame itself plus the weight of roof decking. This loading system corresponds to a roof loading of 50 psf total which a spacing between frames of 20 ft.

The design of the wedge-beam frame was a cut-and-try process. In conventional design the only variable is the size of the rolled section; that is, a section must be chosen which furnishes sufficient section modulus and area to meet the requirements of the loading system without overstressing the steel. In wedge-beam design, there may be two variables, since one method by which a wedge-beam may be constructed is by splitting a rolled wide-flange section along its web and rewelding together the two segments thus formed, as illustrated in Fig. 3. The two variables are the size of the rolled section from which the wedge-beam is to be made (the "parent" section) and the taper

or slope of the cut. It is readily seen that a sharper cut will give a deeper cross-section at the haunched end of the beam, thus providing a greater section modulus at this point, and a more shallow cross section at the small end. In this design, a 4:1 cut was used as a basis, the 4:1 cut meaning that the resulting haunched end of the wedge-beam would be four times as deep as the small end. After preliminary calculations, a 16WF40 rolled section was decided upon, and this section was cut at the 13 inch and 3 inch points, as shown in Fig. 3, resulting in the girder and column members shown in the same figure.

Each girder member would be joined to a column member to form one-half of a symmetrical three-hinged, or articulated frame. The two half frames would then be joined by a hinged connection at the crown. The resulting structure is statically determinate, and when loaded as previously explained gives a horizontal thrust of 16.67 kips and a vertical reaction of 20 kips at each base pin. The moment diagram is shown in Fig. 2, and the thrust and reaction computations are shown in Part IV.[1]

This structure was then analyzed to see if the maximum steel stresses were approximately equal to the design limits. An analysis was made of the combined stresses of direct and flexural loading at the critical knee sections D' and D", Fig. 4.[2] The results of the analysis (See Part IV) showed

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[1] The analytical work described throughout this section will be found in detail in Part IV, Computations and Experimental Data.
[2] Griffiths. Single Span Rigid Frames In Steel.

that both the girder and column were underdesigned. A three or four per cent underdesign would generally be permitted in the field,[1] but in actual design work the column, overstressed by nine per cent, would be redesigned. However, in wedge-beam framing a mere sharpening of the cut would provide the required properties without any change of "parent member". The nine per cent underdesign was permitted to stand without change, since the prototype was not to be built and it was anticipated that a sharpening of the slope of the wedge-beam would increase the difficulty of model construction. The small ends of the column wedge members in the model would have been of prohibitively small dimensions had the slope of the column wedge members been increased.

In order to simplify reasonably the design of the models, the following assumptions were made regarding the loading of the prototype: (1) Only static loads act. Wind and other forces of dynamic origin were not considered. The static forces considered were two, namely, gravity and elasticity. (2) There was no accelerated movement of any model components; hence inertial forces were not considered.

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1. The justification for this statement may be found in the example design computations in Griffith, Single Span Rigid Frames In Steel.

3-3. Computation of Deflection and Deflection Angles.

As previously mentioned, the linear deflection at the crown hinge and the angular deflections at the base pins and the crown hinge were computed theoretically. The first method used in these computations (virtual work) is of very general application and need not be derived. The results were checked by the use of formulae proposed by Mr. Amirikian; the numerical discrepancy between the two methods was so small as to be negligible in the case of the angular rotation at the base pins, and amounted to only about 0.4% in the case of the angular rotation at the crown hinge. The small discrepancy in the latter case was entirely of arithmetical origin. Actually the computations in the two methods are in essence the same; this fact in itself argues the validity of both methods, since they were arrived at by dissimilar derivations. The derivation of the Amirikian formulae follows.

Development of Deflection Angle Formulae
after Amirikian

ALL SYMBOLS REFER TO FIG. 7

General Case - Unsymmetrical Loading

deflection angle at crown = θ_{BA}

deflection angle at base pin = θ_{AB}

m_h = moment due to horizontal unit load at crown

m_v = moment due to vertical unit load at crown

$$\Delta_1' = \int_A^B \frac{My'ds}{EI_x} \qquad \Delta_2' = \int_C^B \frac{My'ds}{EI_x}$$

$$\Delta_v = \frac{1}{2\cos\alpha}\int_A^C \frac{My'ds}{EI_x} = \int_A^C \frac{Mm_v ds}{EI_x}$$

$$\Delta_h = \int_A^C \frac{Mm_h ds}{EI_x}$$

$$\theta_{BA} = \frac{1}{l'}\int_A^B \frac{Mx'ds}{EI_x} + \frac{\delta_1/\cos\alpha}{l'/\cos\alpha} = \frac{1}{h}\int_A^B \frac{Myds}{EI_x} + \frac{\Delta_h}{h}$$

$$\theta_{AB} = \int_B^A \frac{Mds}{EI_x} - \theta_{BA}$$

The expressions for θ_{BC} and θ_{CB} are identical to the above except for integration limits.
For application of these formulae see Table IV, Part IV.

3-4. Semi-Flexible Welded Connection. As previously stated, the second model was exactly similar to the first except that a welded steel strap connection was substituted for the true hinged connection at the crown. This semi-flexible connection for the prototype was designed as two plate steel straps, one to be fillet-welded to the front side of the two adjoining webs and the other to the back side. See Fig. 8. These straps were deliberately made small enough so that they would be stressed beyond the elastic limit into the plastic range. The rectangular fibre stress distribution shown in Fig. 8 was assumed; this distribution is a limit condition and is not actually reached at any time under loading. 33 ksi was used as a conservative value for the yield point of mild structural steel, and hence as the value of the fibre stress ordinates in the stress distribution diagram of Fig. 8.

The design criteria for the welded straps were (1) the vertical shear at the crown caused by unsymmetrical uniform loading of half the span of the frame, and (2) the compression in each strap caused by the horizontal thrust to be transmitted. Computations for each criteria are shown in Part IV. It was found that the compression ruled the design; each strap was treated as a strut, its L/R ratio was computed and the allowable compressive unit stress determined. Two straps, 2" x ¼" in cross section, were found to be sufficient. The unsupported length or gap between adjoining girders was taken as ⅛", which

is sufficient to prevent any interference between the two girders when they deflect under loading. The total length of the straps was not designed; this length must be long enough to allow sufficient fillet welding to handle the horizontal thrust.

The moment resistance of this particular strapped connection, as computed in Part IV, was 1.375 ft-kips.

A good approach to an understanding of the properties of a welded connection acting in a wedge-beam frame may be gained by initially considering two extreme cases of such a connection. One limiting case is illustrated in Table V; this case involves no strap whatsoever, the abutting ends of the girder members merely being welded together throughout their depth. This connection is a rigid one and makes the frame statically indeterminate to the first degree. The horizontal thrust was considered as the redundant reaction and was solved for by the general method of indeterminates. Virtual work was used for the computation of the necessary deflections. It was found that the moment sustained at the crown by this connection was 36.9 ft-kips. and the moment at the knee was 163.1 ft-kips.

The other limiting case has previously been discussed and is illustrated by the use of a true hinged connection, or pin, at the crown. This connection can take no moment and results in a loaded frame with zero moment at the crown and 200 ft-kips at the knee. It is seen that the plastic welded strap designed in this section is intermediate in nature between these two extremes

but that it is apparently much closer to the case of the pinned connection. As explained in the Introduction, the purpose of this thesis is to determine whether the elastic action of the frame when connected with a strap which deforms plastically is substantially the same as its action when connected at the crown with a pin. If this is so, then the theoretical derivations for true hinged wedge-beam framing will hold valid in the case of the relatively inexpensive welded strap connection and the field of application of wedge-beam framing will be enlarged.

There is an additional aspect of a welded strap connection which is worthy of serious study, although it cannot be covered in this work. An examination of the two extreme cases heretofore described will demonstrate that the sum of the moments existing under load at the crown and at the knee is in any case 200 ft-kips. That is, the welded strap will take whatever moment it can until it yields and the remainder of the 200 ft-kip moment will be thrown back to the knee of the frame. It is seen that the welded strap can be over-designed to varying degrees so that it takes any portion of the 200 ft-kips up to the limit of 36.9 ft-kips, although any appreciable overdesign will, of course, destroy the validity of the Amirikian theoretical work for wedge-beam framing. However, as long as such deliberate overdesign does not exceed the moment resisting properties of the small ends of the girder members immediately adjacent to the crown, the overdesign could be utilized by a proper extension

of theory to effect even further economies in steel framing. The girder and column members of a frame are designed by a computation of stresses at critical sections of the knee (See Fig. 4) and it is in this same region that the stresses are reduced by an appropriate overdesign of the welded strap. To recapitulate, the experimentation in this work was carried out to determine whether the strap actually bends into the plastic range substantially as is shown theoretically in Fig. 6. The strap acting in this fashion would demonstrate only the small moment resistance of 1.375 ft-kips (computed in Part IV) and probably would not materially alter the elastic action of the frame from its action when a true hinge is acting at the crown.

2-6. Construction of the Model: The Laws of Similitude. Symbols used in this section are defined as follows:

L	linear dimension
A	area of cross-section
I	moment of inertia
M	moment of a force
E	modulus of elasticity
W	weight or gravitational force
F	force in general
w	weight per unit volume
n	linear scale reduction factor
e	unit strain

These symbols when unmodified refer to the prototype; the subscript "1" indicates a function of the model.

A linear scale reduction factor is first assumed and other relationships of model to prototype are mathematically deduced from this factor. The two fundamental principles upon which this deduction is based are that the model and prototype must

be geometrically and mechanically similar.[1] In order to achieve geometric similarity, homologous linear dimensions of the model must be proportional to those of the prototype. In order to achieve mechanical similarity, homologous forces must have a fixed ratio to each other.

There are two forces acting on the frame:[2]

(1) The external forces due to gravity's action on the loading, expressed by

$$F = W \qquad (1)$$

(2) The internal elastic forces in the frame members, expressed by

$$F = AEe \qquad (2)$$

Then, because of mechanical similitude

$$\frac{F}{F_1} = \frac{AEe}{A_1E_1e_1} = \frac{W}{W_1} \qquad (3)$$

But geometric similitude requires that unit strains in the two systems be equal; hence

$$\frac{W}{W_1} = \frac{AE}{A_1E_1} \qquad (4)$$

A linear reduction of n would of course result in a reduction of area of cross-section by n^2; however, with special reference to a structural frame, it is not necessary to limit the area reduction to this value, although it is desirable to keep the

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[1] Beggs, Davis and Davis: Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge. (Berkeley, Calif.1932)p.

[2] Beggs, Davis and Davis. Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge. pp. 67-71.

reduction as some function of n^2. It has been found advantageous to determine the area reduction in conjunction with the force reduction, the latter being assumed as

$$\frac{W}{W_1} = n^2 \qquad (5)$$

Substituting in equation (4) to obtain the area reduction factor

$$\frac{A}{A_1} = n^2 \times \frac{E_1}{E} \qquad (6)$$

A moment of a force may be expressed in terms of a force times a linear dimension

$$\frac{M}{M_1} = \frac{FL}{F_1L_1} = n^2 \times n = n^3 \qquad (7)$$

A comparison of two similar members should contain the terms for moment of inertia and modulus of elasticity, since the stiffness of a flexural member is dependent upon both of these terms. The elastic curve of a beam is

$$M = EI \times \frac{1}{R} \text{ or } I = \frac{MR}{E} \qquad (8)$$

where R is the radius of curvature of the beam. R is a linear dimension and the ratio of Rs may be expressed in terms of the linear reduction factor n.

$$\frac{I}{I_1} = \frac{MR}{E} \; \frac{E_1}{M_1R_1} = n^3 \times n \times \frac{E_1}{E} = n^4 \times \frac{E_1}{E} \qquad (9)$$

Weight may be expressed in the force equation

$$W = wAL \qquad (10)$$

Then

$$\frac{W}{W_1} = \frac{wAL}{w_1A_1L_1} = n^2 \qquad (11)$$

Substituting for $\frac{A}{A_1}$ and $\frac{L}{L_1}$

$$\frac{w}{w_1} \times n^2 \times \frac{E_1}{E} \times n = n^2 \qquad (12)$$

or

$$\frac{w}{w_1} = \frac{E}{E_1} \times \frac{1}{n} \qquad (13)$$

The last formula giving the density relationship is redundant in a sense because if the load and area reduction factors are followed rigorously the density reduction is automatically obtained in the model design. The derivations of the laws of similitude given above are relatively simple, and the deduced laws apply only to static models.

Strictly, all of the above laws should be used when a model structure is proportioned from a prototype design. However, in this work it was virtually impossible to proportion a model using both the reduction laws for moment of inertia and for area. Since the model was to be tested in flexure, the former term was manifestly more important and the moment of inertia was reduced exactly according to equation (9) above. Thus equation (6) above (and hence equation (13) also) were the only laws of similitude not followed strictly in the model design.

A model designed in adherence to these laws of similitude should show when loaded angular deflections and unit linear deflections equal to those of the loaded prototype.

3-6. Choice of Material for Model Construction. Duralumin, brass, celluloid, and steel have all been used in models similar to the one designed in this work. Since this thesis was concerned with the study of a welded semi-flexible connection, the selection of steel as a model material was almost mandatory. Too many uncertainties would have been introduced by attempting to weld materials other than steel, since the prototype frame was steel construction. Once this choice was made, the selection of the direct method of model analysis (whereby the model is loaded in a manner similar to the prototype loading) followed naturally, since the indirect method (in which the model loading bears no direct relation to the prototype loading) generally involves the use of materials of the relative stiffness of celluloid.

3-7. Linear Reduction Factor. It was anticipated that a relatively large model would be necessary to provide sufficient space in the region of the crown for the welding of the semi-flexible strap. In fact, a straight linear reduction of the welded strap previously designed for the prototype by any reasonable linear reduction factor would result in a model strap too small to be fabricated in the model. It was desired to obtain a strap connection in the model of moment resistance proportional to the moment resistance of the prototype strap connection. In order to obtain this proportional relationship it was decided to build the largest model convenient for construction and experi-

mentation. The figure of n = 8 was finally chosen, resulting in a model of 40/8 or 5 feet in span and 12/8 or 1½ feet in height. The smallest welded strap that could be fabricated, even using a model of this size, still offered a moment resistance proportionally greater than the prototype strap, although the proportional relationship was substantially attained.

A larger model also reduces the effect of secondary stresses and the effect of small inaccuracies in construction.

The results obtained from model analysis depend upon the scale of the model. A deflection measurement observed from a small model would not be exactly equal to the corresponding deflection measurement observed from a large model. This phenomenon is known as scale effect. It is one of the uncertainties of model analysis and is a subject worthy of investigation each time such an analysis is attempted. An investigation of scale effect was considered outside the scope of this thesis; however, the construction of a smaller model than the one described above (n = 20) was contemplated but not consummated. The design of this smaller model is included in (Tab. VIII) for reference. The table gives the depths of the model numbers at the centers of the various segments. This smaller model was also designed of steel.

3-8. Cross Section of Model Members. In model analysis where direct loading is used, lateral buckling (deformation in directions at right angles to the action of gravity forces) is generally a problem. Later stability in model members was achieved in this work by simulating a rolled wide-flange section. This type of section complicated the construction of the models, since it was necessary to weld thin plate steel to simulate it. However, the extra stability and similitude obtained was considered worth the complication.

The smaller model (n = 20), which was proportioned but not constructed, was designed for a rectangular cross section, since its smaller span limited the amount of buckling expected.

The governing factor in the design of the cross section of the model members was the proportional reduction of the moment of inertia from the cross section of the prototype. This reduction was by the fourth power of the linear reduction factor n, as derived in the previous section and presented in Table VI for the larger models. It was necessary to proportion the models so that they would present a moment of inertia equal to I_m, Table VI, at the center of the proper segment as shown in the same table . (I_p was obtained from Fig. 5.) This was done by designing the web members of 11 gauge plate steel and of dimensions reduced exactly by eight from the corresponding web dimensions of the prototype (See Table VII). The moment of inertia furnished by each web alone is represented as I_{web}.

Col. (6), Table VII; the remainder of the total I required in the model (I_m) was furnished by the flange sections, which were of 16 gauge steel welded to the web sections. In order to furnish exactly the proper increment of moment of inertia to bring the total I up to I_m, it was necessary to taper the width of the flanges from a minimum of 0.160 in. in the region of the knee on approximately a straight line variation to a maximum of 0.677 in. at the pins.[1] This particular method of designing the model also made it possible to compensate for deviations in actual thickness of the steel plates from their catalogue thicknesses by varying the flange areas accordingly. The larger models, as designed, presented a moment of inertia at all sections exactly equal to $1/_{n}4$ times that of the prototype, although it was doubtful if construction procedures maintained this accuracy, even though all precautions within the limits of time and money available were taken in building the models.

Since the smaller models (n = 20) were of rectangular cross section, it was only necessary to compute the reduced moment of inertia at each of the various segments and from this calculate directly the depth of the cross section at the centers of the various segments. An additional arbitrary factor m was applied to both the laws of similitude for weight

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1. It should be mentioned at this point that, although this thesis deals only in wedge-beams of tapered depth and constant flange width in the prototype, Mr. Amirikian has extended his theoretical wedge-beam derivations to include the case of wedge-beams with tapered depth and tapered flanges.

loading and for moment of inertia, as shown in Table VIII. This factor m has sometimes been called a "slice factor"; its use is a matter of convenience and does not invalidate the laws of similitude. As previously mentioned, these smaller models were not actually constructed, although they may be built at some later date in an extension of this work.

2-9. Mounting of Model. To simplify the lateral bracing of the models, it was elected to mount the model horizontally. (See photographs.) A plywood sheet, 8' long by 4' wide by 3/4" thick, was secured atop two tables which had been placed side by side. A 2 x 4 was attached at each edge of the plywood to steady it on the tables and to prevent its overturning under the applied load. It was believed that the column base pin reactions of approximately 300 lb. and the thrusts of approximately 260 lb. would be excessive for the 3/4" plywood to resist in bearing. Hence a ¼" steel base plate, six inches square, was provided to take the bearing from each column pin. Each of these two steel plates transmitted the reactions and thrusts to the plywood through eight wood screws. The base pins were anchored in each base plate by the use of small angle sections welded atop the plates and drilled to receive the ½" steel rods which served as the base pins.

To prevent frictional effects between the model sections and the plywood, one-half inch ball bearings were placed at several points under the webs of the model members. These ball bearings rolled on thin glass sheets placed on top of the plywood.

For lateral support, weights were placed as needed on top of the model members directly over the ball bearings.

3-10. Welded Strap for Model. It was stated previously that the welded strap connection for the model could not be reduced in any fashion from the design of the welded strap for the prototype, since such reduction would result in a strap too small to be fabricated in the model construction. A clearance or gap of ¼" between adjacent girder members at the crown was decided upon; this value was well in excess of the computed minimum distance for avoiding any interference between the two members when the deflect under loading. Three trial straps were computed (See Part IV) and the moment resistance of each of these determined by the stress distribution theory illustrated in Fig. 8. According to the laws of similitude, the moment resistance of this strap should be $1/_{n}3$ times that of the prototype. The first two trial straps, of thickness 1/8", offered a resistance considerably in excess of this value; the third trial strap, of thickness 1/16", offered a resistance of about twice this value. All computations were based on a pair of identical straps, one welded in front of and the other behind the web. It was finally decided to use but one strap in front of the web. Although this caused a slight eccentric loading of the one strap, it was believed that the advantage gained by having a welded connection yielding a moment resistance properly

proportional to the prototype resistance outweighed the disadvantage of eccentric loading. Therefore the strap connection used consisted of one small metal plate, 1/16" thick, ¼" deep, and approximately 2¼" in length, welded across the webs of the adjacent girders at the crown.

3-11. Loading of Model. In accordance with the laws of similitude, the total load on the model was reduced by the factor n^2 from the total load on the prototype. Since linear span was reduced by n, the resulting reduction in linear load was n^2/n or n. This gave for the model a total load of $\frac{40000}{(8)^2}$ or 625 lb. and a linear load of $\frac{1000}{8}$ or 125 lb. per foot of span. Such a loading appears excessive; yet the selection of a larger model, for reasons heretofore given; necessitated a loading of this magnitude. The arbitray factor m, mentioned in Section 3-8, could have been applied to lessen the loading on the model, but it also would have been necessary to apply it to the reduction of the moment of inertia for the model, resulting in a lesser moment of inertia of cross section at each segment of the model. The smallest moment of inertia in the model (at the small ends of the wedge beams) was 0.01379 $in.^4$ without the use of the factor m, and any lessening of this value would have resulted in a prohibitively small cross section at the small ends of the beams. Therefore it was judged impracticable to lessen the required loading by use of the arbitrary factor m.

Since it was not feasible to simulated uniform loading on the model, it was decided to apply ten concentrated loads at intervals of six inches symmetrically placed about the mid-point of the frame. Each concentrated load, therefore, was $^{625}/_{10}$ or 62.5 lb.

It was necessary to apply the loading in small increments in order to obtain data for plotting load vs. deflection curves. The loading problem thus presented was to obtain a material of total weight 625 lb. which could be applied in small increments, and of sufficient density so that excessive volume was not required. The first loading considered was large calibrating weights, but these were unobtainable in sufficient quantity. Water and sand were discarded because of the large volumes required in the limited space available. Mercury was considered but rejected because of the prohibitive cost, although it would have been an ideal loading agent. Finally a sufficient quantity of steel punchings were obtained, weighing approximately one-half lb. each.

It was decided to apply the loading to the frame by bracing $^{3}/_{32}$" braided steel cable in loops around the girder sections at the ten loading points. Each loop was prevented from wandering from the loading point by small shallow steel channels welded at the loading point and through which the cable was threaded. The lengths of steel cable were led over pulleys mounted at the edge of the plywood and thence to steel buckets which acted as containers for the steel punchings. It is seen that the load

was applied to the top flange, simulating the loading of the prototype.

The pulleys were 2" outside diameter fibre ball bearing pulleys of the aircraft control type, and were assumed frictionless. The buckets were made of 11 gauge steel rolled to a five-inch diameter and tack welded; their bottoms were 11 gauge plate, and heavy wire handles were attached. A scale was used to weigh out desired amounts of steel punchings.

3-12. Construction of Members. It was first attempted to attach the 16 gauge flange plates to the 11 gauge webs by intermittent fillet welds on the inside of the flanges. A trial section was thus welded; the result, in spite of skip welding and other precautions, was a lateral distortion of the web plate.

It was next attempted to attach the flange plates to the web plates by drilling $^9/_{64}$" holes in the flange plates at regular spacing and plug welding through these holes to the edge of the web. The welds did not penetrate on the trial sections and the joining was unsuccessful. This method was modified by countersinking the holes in an attempt to secure weld penetration, but the effort was again unsuccessful.

Finally slot welding was tried and a successful joining attained. The flange plates were split in halves and the two halves were simultaneously welded to the web by a continuous slot weld to form each flange. This method has the outstanding advantage that it does not change the cross section of the beam by any deposition of weld metal.

A Lincoln DC welding generator was used; 22 volts and 90 amperes current were used, and the rod was a 3/32" coated electrode, AWS E 6012.

After the flange sections were welded to the web, they were milled down to give the tapered width flange previously discussed.

3-13. Fabrication of Models. The girder members and column members, constructed as described in the last paragraph, were joined by butt welding the top edge of each column to the underside of the corresponding girder. Dimensional accuracy in this welding process was assured by the use of steel jigs especially constructed for the purpose. Each member was pinned to the jig so that it was in its correct position relative to the member to which it was being joined, and so that it could not creep from that position under the influence of welding stresses.

A deep rectangular nut, drilled out to receive a ¼" pin, was welded to each column at its lower end. This nut, when connected by a pin to the steel base plate previously described, provided a true hinged connection of zero moment resistance at the base of each column. This base detail was identical in the articulated model and in the semi-articulated model.

To form the hinged crown connection in the articulated model, the web of one of the adjoining girders was extended by welding a plate steel tongue to it. This steel tongue was

drilled to receive a ¼" pin. Two flanking sections of plate steel were welded to the web section of the other girder; these flanking sections were also drilled for a ¼" pin. When this model was erected for testing, the tongue section was inserted into the gap between the flanking sections, the pin holes were lined up, and a small removable pin was used to join the two girders together.

The strap which was used for the semi-flexible connection at the crown of the semi-articulated model has been previously described as 1/16" thick, ¼" deep, and approximately 2¼" long, and the reason for using one strap in the connection instead of two has been shown. (See Section 3-10, Page 25). This strap was welded to the front of the web of each adjoining girder by an around-the-end fillet. The welds were of sufficient strength to remain absolutely rigid, so that all bending and yielding took place in the strap proper.

Consultation of the included photographs should clarify the description of model construction.

EXPERIMENTATION

3-14. Instrumentation. The apparatus used for measuring deflections consisted of steel pointer arms, Ames deflection dials, and the mountings for the dials. The steel pointer arms were made of drill rod stock; they were 12" long and accurately threated on one end so that they could be rigidly screwed into nuts which were tack welded at the proper places

on the column flanges and beam flanges. (See Fig. 9). One pointer arm was connected near the crown, and one was connected near each base pin, resulting in a total of three pointer arms.

The basic formula used in computing deflection angles was

$$s = r\Theta$$

where s represents linear length of arc, r represents radius of rotation, and Θ is the subtended angle in radians. For the small angles involved in this work, s was very closely equal to the linear length of chord.

The Ames dials were mounted so that their plungers made contact with the pointer arms at a point approximately ten inches from the centers of the pins. Ten inches, then, was approximately the value of r in the formula above; any deviations from ten inches were of course accurately measured and the corrected radii used in computations. The cylindrical surface of each pointer arm was beveled down to a flat surface where contact was made with the plungers of the Ames dials in order that the plungers would not slip off of the arms.

The Ames dials measured the deflections of their plungers to thousandths of an inch. The readings of the Ames dials, then, represented linear length of chord and were used for s in the formula above. Thus s and r were measured at three points where the model members deflected angularly under load, and the formula was used to compute the angular deflections at these points.

Small lengths of aluminum angle sections were used to mount the Ames dials to the plywood. The downstanding legs of the angles were screwed to the plywood, and the dials were attached to the upstanding legs. The dials were mounted so that there was spring tension on the plungers initially, or before loading was applied to the model. This was done to take up any bending of the metal pointer arms before actual experimentation was begun.

The linear deflection at the crown was measured directly by mounting an Ames dial with its plunger in contact with the underside of the flange of one girder immediately to the right of the pin.

3-15. Experimentation with Articulated Model. The ten buckets which were to be used as weight containers for loading the model were weighed individually. The heaviest of the ten was found to weigh 8.8 lb.,[1] and the other buckets were ballasted to bring their weights up to this amount. Next ten portions of the scrap steel punchings were measured out so that each portion weighed 21.2 lb.; ten more portions were measured out to 15 lb. each; ten more portions were measured out to 17.5 lb. each. All of these portions of steel punchings were placed in the immediate vicinity of the model so that they would be readily available for

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[1] It was decided that weight measurements to the nearest 0.1 lb. were in keeping with the accuracy of model construction.

loading, and each of them was placed so that there would be no confusion as to the weight of any particular portion during experimentation. To load the model, these portions of steel punchings were placed manually in the buckets, starting with the extreme outside buckets at each end of the model and proceeding toward the center buckets. Symmetrically opposite buckets were loaded simultaneously in small increments.

Thus the load points on the load vs. deflection curve to be plotted for the articulated model were 0 lb., 8.8 lb., 30 lb., 45 lb., and 62.5 lb. for each bucket, or 0 lb., 88 lb., 300 lb., 450 lb., and 625 lb. for total loading on the model. This corresponded to a simulated uniform loading of 0 lb. per foot of span, 17.6 ppf, 60 ppf, 90 ppf, and 125 ppf on the model. Multiplying each of these uniform load points by the linear reduction factor n = 8 gave the corresponding uniform loading on the prototype. 125 x 8 = 1000 ppf = 1 kpf = design load for prototype.

3-16. Results for Articulated Model. At a total loading of about 602 lb. on the model, or 96.5% of the total load to be applied, the section of the model to the right of the crown hinge apparently buckled slightly so that its knee joint deflected downward toward the plywood, while the other knee was noticed in a slightly raised position. Loading was stopped and the left knee section was supported manually; when an attempt was made to press this knee section down to its original position, the right half of the frame buckled suddenly and the frame failed by lateral buckling. Therefore the load vs. deflection curve for this model was not plotted to completion, although deflection

readings were obtained from the Ames dials immediately before failure and the curve was substantially complete.

The failure of this model was explained by the lack of real lateral support. The prototype was designed assuming continuous lateral support along the length of the girder provided by the steel roof planking and other longitudinal members of the structure. The flange areas of the girder members in the prototype were constant with length. In the case of the model, this continuous lateral restraint was not supplied, and in addition the flange width was varied in order to simulate the moment of inertia of the prototype, thereby depriving the model of one source of lateral stability in the critical region of the knee. The lateral restraint in the experimentation with this model was provided by the ball bearings underneath the web and by steel punchings weighing about 7 lbs. total placed on top of the web directly over the ball bearings. This amount of lateral support was shown to be inadequate in a most graphical fashion during experimentation. Increased lateral support was provided during experimentation with the second (semi-articulated) model, as described in the next section.

The load and deflection data are presented in Table IXa. Deflections measured were linear deflection at crown, angular deflection at crown, and angular deflections at the base of the columns. The tabular data is presented graphically in Graph 5, Angular Deflection at Points B and C, and in Graph 6, Vertical Deflection at Point B. The rotational deflection at Point A,

or at the left base pin, was not plotted because of obvious discrepancies. An examination of dial readings at Point A, Table IXa, will show that angular rotation at this point apparently began in the wrong direction with the initial loadings, but that it changed direction in the process of loading somewhere between 8.8 lb. per bucket and 30 lb. per bucket. This discrepancy was traced to play in the pin connection at A; this play was measurable in thousandths of an inch and destroyed the validity of the first dial readings. However, the first loading forced the base pin into its final position, thus effectively taking up the play or slack, and if the last several dial readings are corrected accordingly they will be found to agree with the results at the other base pin (Point C). The reading at A is not necessary except for purposes of checking, since the model was symmetrically loaded and the base pin deflection could be obtained from Point C. Hence, because of the defective base pin connection, the data from Point A was not plotted for either of the two models.

On Graph 5, the angular deflections at the crown (Point B) and at the Base pin (Point C) for the articulated model are plotted versus load as a solid line. A small rectangular symbol denotes the theoretical computed deflection for each of these cases. A comparison with theoretical results was made by extrapolating these curves the short distance to the design load point, and it was found that the largest per cent deviation was at the crown for rotational deflection (20.0%); the per cent

deviation for rotational deflection at the base pin was 9.6%, and from Graph 6, the per cent deviation for linear vertical deflection at the crown was 11.7%.

3-17. Additional Lateral Support. Since the first model failed at the knee by lateral buckling, brackets were constructed and mounted on the second model to prevent a similar failure. (See photographs). Ball bearings were thereby brought into contact with the top as well as the bottom of the web of the knee sections. This arrangement prevented the failure of the second model.

3-18. Experimentation and Results with Semi-Articulated Model. The amount of loading which would bring the extreme fibers in the welded strap to the yield point was predicted theoretically. It has previously been shown that the moment resulting at the crown of the prototype in the hypothetical case when the abutting ends of the girder members were welded rigidly together would be 35.9 ft-kips. (See Table V). This moment is proportional directly to the load per foot on the prototype; The moment resistance of the welded strap on the model at the yield point was shown to be .002686 ft-kips. By the laws of similitude, .002686 x n^3 = .002686 x 512 = 1.375 ft-kips moment resistance for a corresponding strap on the prototype. Let y kpf = the uniform loading which would cause a moment at a rigid joint at the crown of 1.375 ft-kips; then

$$\frac{y}{1} = \frac{1.375}{35.9}$$

or y = 37.3 ppf for the prototype, or 37.3/8 = 4.66 ppf on the model, or 2.33 lb. in each bucket. Since each bucket alone weighed 8.8 lb., and the time to construct additional apparatus to apply a smaller loading to each wire was not available, it was decided to apply the loading on the semi-articulated model in exactly the same manner as it was applied to the first model (Section 3-15.). What happens in the region of the yield point of the strap is not of particular importance to the results of this investigation; rather an over-all picture of the action of the strap from zero loading to design loading is the main object of the investigation.

The same deflections were measured for this model as were measured for the first model (Section 3-16). The results are plotted as dotted lines on Graphs 5 and 6. It is seen that the welded strap allowed 21.6% more angular deflection at the crown than did the true pin, 13% more linear deflection at the crown than with the pinned model, and 14% more angular deflection at the base pin than with the pinned model.

The entire load was removed from the model and a check was made of the no-load readings (See Table IXb). Point C returned to its initial zero reading; but, as was expected, both the angular and vertical (linear) dials at Point B showed a permanent set. This was due to the overstressing of the strap which caused the metal in the strap to flow plastically. A reloading to the designed model load (125 ppf) showed no appreci-

able change except in the angular deflection at the crown. This is not considered important, since the discrepancy was probably caused by the movement of the pointer arm as explained in Section 3-18.

3-19. Explanation of Discrepancies. The 20.0% maximum per cent deviation between theory and experimentation is not considered excessive. In the 1933 model tests on the San Francisco-Oakland suspension bridge, the observed deflections showed a range of 6% to 12% approximate deviation from theoretically computed values.[1] Possible reasons for the deviations in this work are listed below:

1. The machining or finishing of the model was not done to the expected accuracy; thus the moment of inertia relationship between model and prototype was not rigidly applied.
2. There was an uncontrollable up and down movement of the ends of the pointer arms at the points of contact with the dial plungers. This movement was due to slight twisting of the model members. Thus the plungers rested on a changing surface, and the sensitive Ames dials reflected this. Attempts were made to compensate for this effect.
3. Ten equally spaced concentrated loads were substituted in the model for the uniform load used in theoretical computations for the prototype.
4. There was some slack or play in the base pin connections, as explained in Section 3-16.

These factors are stated as possible explanations for the discrepancies, but any attempt to evaluate quantitatively the effect from each factor would be extremely difficult.

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[1] Beggs, Davis and Davis. Tests on Structural Models of Proposed San Francisco-Oakland Suspension Bridge. P.134.

CONCLUSIONS

3-20. Conclusions in Regard to Object of Investigation. The results of this investigation show that the semi-flexible welded strap apparently offers less moment resistance than does a so-called true hinge at the crown. The pinned connection used in the first model (See photograph) was precisely machined and offered no moment resistance under no load conditions. but under loading local distortions caused moment resistance of greater magnitude than the resistance in the small welded strap. The pin was checked in double shear and double shear bearing and it was found that it was not overstressed. This investigation tends to establish, but of course does not conclusively prove, that local distortions in the region of a pin will often cause greater moment resistance than that obtained from a semi-flexible welded strap of the type designed herein. In regard to the Amirikian wedge-beam theory, the investigation demonstrates that a semi-flexible connection acting in place of a pinned connection will not change the elastic characteristic of the frame appreciably and that it certainly will not change this characteristic in the direction of more moment resistance in the connection. The authors observed that the very small welded strap used in the semi-articulated model took the design load very well, although this strap was designed exactly at the design limit for the ruling factor of compression (Section 4-4), and furthermore the frame itself was underdesigned (Section 3-2).

It is felt that the data from this investigation may be interpreted more advantageously in the light of further data from the succeeding experimentation. Suggestions for such further study are given in the next section.

To reiterate, the following conclusion may be tentatively taken from the data of this investigation: A small welded strap may be used in place of the crown hinge in a three-hinged wedge-beam frame without appreciably changing the deflections of the frame. Furthermore, the deflections apparently increase rather than decrease with the use of the welded strap.

3-21. Suggestions for Further Study. This thesis contains the design of a smaller model with a linear reduction factor of n = 20. (See Table VIII). Construction and experimentation of this model would yield data which would be valuable in the study of scale effect (Section 3-7). Of course the design and construction of several models with varying reduction factors would yield a more complete understanding of scale effect as it applies to this work.

Also it is recommended that a celluloid model be constructed and analyzed by the Beggs Deformeter technique. The celluloid model could be locally reduced or cut down in the region of the crown to simulate the stiffness of the strap relative to the stiffness of the girder members, although a large model would be needed.

If a method could be devised for successfully removing and rewelding the small strap connection on the second model used in this work, it is suggested that experimentation be conducted with different straps acting at the crown. These straps would be increased by small increments from the size of the welded strap used in this work.

Perhaps more applications of semi-flexible welded straps acting as valid substitutes for pins could be found in the general field of structural engineering.

IV. Computations and Experimental Data

4-1. **Design of Prototype.** (See Fig. 2).

Vertical reactions at A and C

$V = \frac{1}{2} \times 1 \times 40 = 20^{k}$

Horizontal thrusts at A and C

$M_B = 0$ (clockwise moments assumed positive)

$M_B = +(20 \times 20) - (H_A \times 12) - (1 \times 20 \times \frac{20}{2}) = 0$

$H_A = +16.67^{k}$ (acting toward right)

A 16 WF 40 with a 4:1 cut was chosen. (See Page 9)

Critical sections D' and D" were checked for combined loading. (See Fig. 4)

Section at D'

Moment

$$
\begin{aligned}
-16.67 \times 12 &= -200.00^{fk} \\
-1.0 \times \frac{(1.083)^2}{2} &= -0.59 \\
+20 \times 1.083 &= \underline{+21.66} \\
&\quad -178.93^{fk}
\end{aligned}
$$

Req'd $S = \frac{M}{f} = \frac{12 \times 178.93}{20} = 107.4$ in.^{3}

$I_{D'} = 1293$ in.^{4}

$A_{D'} = 2 \times 7 \times 0.503 + 2.3.91 \times 0.307 = 7.04 + 7.34 =$ 14.38 in.^{2}

Assume continuous lateral bracing

$F_A = 17.00$ ksi.; $F_B = 20.00$ ksi.

$f_B = \frac{Mc}{I} = \frac{178.93 \times 12 \times 12.46}{1393} = 19.19$ ksi.

$f_A = \frac{16.67}{14.38} = 1.16$ ksi.

$\frac{f_A}{F_A} + \frac{f_B}{F_B} = \frac{1.16}{17.00} + \frac{19.19}{20.00} = 0.0682 + 0.958 = 1.026$

(2.6% underdesign)

Section at D"

Moment

$- 16.67 \times 10.92 = - 182.0$ fk

Req'd $S = \frac{12 \times 182.0}{20} = 109.2$ in.^{3}

$I_{D"} = 1303.5$ in.^{4}

$A_{D"} = 7.04 + 7.12 = 14.16$ in.^{2}

Assume lateral bracing at 2' intervals

$r_{D"} = (I/A)^{\frac{1}{2}} = \left(\frac{1303.5}{14.16}\right)^{\frac{1}{2}} = 9.6$ in.

$\left(\frac{l}{r}\right)_{D"} = \frac{2 \times 12}{9.6} = 2.50$; $F_A = 17.00$ ksi.

$\frac{ld}{bt} = \frac{2 \times 12 \times 24.20}{7.00 \times 5.03} = 164.5$; $F_B = 20.00$ ksi.

$f_A = \frac{20.00}{14.16} = 1.412$ ksi.

$f_B = \frac{182.0 \times 12 \times 12.10}{1303.5} = 20.2$ ksi.

$\frac{f_A}{F_A} + \frac{f_B}{F_B} = \frac{1.412}{17.0} + \frac{20.2}{20.0} = 0.083 + 1.01 = 1.09$

(9% underdesign)

Web Shear at A

Allowable shear = 13 bt = 13 x 6 x .307 = 23.95 ^{k}

Actual shear = 16.67 ^{k} SAFE

Section moduli furnished compared with section moduli required along girder. (See Table I and Graph I).

Section moduli furnished compared with section moduli required along column. (See Table I and Graph 2).

4-2. Computations of Deflections.

By Virtual Work

At crown hinge (B) (See Table II)

At base pins (A & C) (See Table III)

By Amirikian Method (See Table IV)

4-3. Design of Welded Straps to Replace Crown Hinge.

(See Fig. 8 and Section 3-4, Page 13)

(a) Vertical Shear

(1) Assume bent weightless and a live load of one kip per foot applied on left half of girder only.

$M_c = -(1x20)(20+10) + V_A (40) = 0$

$V_A = 15^k$

$V_B = 20 - 15 = 5^k$

(2) Assume girder weight of 40 pounds per foot and live load (on left half of girder only) of 960 pounds per foot.

$M_c = -(40x40)(20) - (960x20)(20+10) + V_A(40) = 0$

$V_A = 15.2^k$

$Y_B = 20 - 15.2 = 4.8^k$

Assumption (1) gives maximum shear at B.

According to Section 21(a), AISC Specifications, the minimum shear for which a welded connection can be designed is 10 kips.

Allowable shear stress = 13 ksi.

Assume one strap on each side of web and a depth of 2".

Req'd strap thickness = $\frac{10}{13 \times 2 \times 2} = 0.192$ in.

(b) Compression

Assume two straps with following dimensions:

$d = 2"$

$t = \frac{1}{4}"$ (See Fig. [illegible])

$l = \frac{1}{2}"$

Least radius of gyration = $r = (I/A)^{\frac{1}{2}}$

$I = \frac{1}{12} dt^3 = \frac{1}{12} (2)(\frac{1}{4})^3 = \frac{1}{384}$ in.^{4}

$A = dt = 2 \times \frac{1}{4} = \frac{1}{2}$ in.^{2}

$(I/A)^{\frac{1}{2}} = (\frac{2}{384})^{\frac{1}{2}} = \frac{1}{13.86}$ in.

$\frac{l}{r} = {}^{\frac{1}{2}}/{}^{1}/_{13.86} = 6.93$

Allowable compressive stress

$f = 17{,}000 - 0.485 (\frac{l}{r})^2 = 16977$ psi. or 16.98 ksi.

Req'd Area = $\frac{16.67}{2 \times 16.98} = 0.491$ in.^{2} per strap.

Furnished Area = $dt = 2 \times \frac{1}{4} = 0.50$ in.^{2} per strap

(c) Straps selected

2 straps

$d = 2''$

$t = \frac{1}{4}''$

$l = \frac{1}{2}''$

(d) Formula for moment resistance of straps for prototype (See Fig. 8)

(1) Derivation

Assume stress diagram as shown in figure.

One strap

$$M_R = 2(S_y\ A/2 \times d/4) = S_y \frac{Ad}{4} = \frac{33}{4}(dt)d \text{ ik.}$$

Both Straps

$$M_R = 2 \times 33/4\ d^2t \times \frac{1}{12} = 1.375\ d^2t \quad \text{fk.}$$

(2) Application to selected straps.

$$M_R = 1.375\ (2)^2\ (\tfrac{1}{4}) = 1.375 \quad \text{fk.}$$

4-4. Design of Model.

(a) Application of laws of similitude.

(See Section 3-5, Page 16, and Section 3-7, Page 20)

$L/L_1 = n = 8$

Beam length of model $= 40/_8 = 5$ ft.

Column height of model $= 12/_8 = 1.5$ ft.

$W/W_1 = n^2 = 64$

$W_1 = \frac{40{,}000}{64} = 625$ lbs.

$\frac{I}{I_1} = n^4 = 4096$ (See Section 3-8, Pg. 22, Table VI, and Table VII)

(b) Weld design - flange to web section.

Horizontal Shear Check (See Fig. 9)

Pt. D'_1 (See Fig. 4)

$$H_{D'_1} = \frac{VQ}{I} = \frac{312.5 \times 0.146 \times 1.56}{0.344} = 20.6 \text{ ppi}$$

Point of application of one of loads nearest Pt. B.

$$H_X = VQ/I = \frac{62.5 \times 0.0437 \times 0.500}{0.0268} = 51.0 \quad \text{ppi}$$

Since a continuous slot weld was selected (See Section 3-12, Page 28) and the horizontal shear was so low, this weld design need not be continued.

(c) Welded strap connection (See Section 3-7, Page 20, and Section 3-10, Page 25)

Compression

First trial strap (d = 3/8"; t = 1/8"; l = 1/4")

$$I = 1/12\ (1/8)^3\ (3/8) = \frac{3}{12 \times 64 \times 64} \quad \text{in.}^4$$

$$A = 3/8 \times 1/8 = 3/64 \text{ in.}^2$$

$$r = (I/A)^{\frac{1}{2}} = 1/27.7 \text{ in.}$$

$$l/r = 27.7/4 = 6.92$$

$$f_{c_{allow.}} = 16.98 \text{ ksi.}$$

$$P = H_A = \frac{1}{18}(62.5)(3 + 9 + 15 + 21 + 27) = 260.5 \text{ p}$$

or 0.2605 k

$$A_{reg.} = \frac{0.2605}{16.98} = 0.01535 \text{ in.}^2$$

$$A_{actual} = 2(3/8)(1/8) = 0.094 \text{ in.}^2 \quad \text{(too much)}$$

Second trial strap ($d = \frac{1}{4}"$; $t = 1/8"$; $l = \frac{1}{2}"$)

$$I = \frac{1}{12}(1/8)^3(1/4) = \frac{1}{12 \times 64 \times 32} \text{ in.}^4$$

$$A = (1/4)(1/8) = 1/32 \text{ in.}^2$$

$$r = (I/A)^{\frac{1}{2}} = 1/27.7 \text{ in.}$$

$$l/r = \frac{27.7}{4} = 6.92$$

$$A_{req.} = \frac{0.2605}{16.95} = 0.01536 \text{ in.}^2$$

$$A_{actual} = 2(1/4)(1/8) = 0.0625 \text{ in.}^2 \text{ (too much)}$$

Third trial strap ($d = \frac{1}{4}"$; $t = 1/16"$; $l = \frac{1}{2}"$)

$$I = \frac{1}{12}(1/16)^3(1/4) = \frac{1}{48 \times (16)^3} \text{ in.}^4$$

$$A = \left(\frac{1}{16}\right)(1/4) = \frac{1}{64} \text{ in.}^2$$

$$r = (I/A)^{\frac{1}{2}} = 1/55.4 \text{ in.}$$

$$l/r = \frac{55.4}{4} = 13.85$$

$$f_{c_{allow.}} = 16.91 \text{ ksi}$$

$$A_{req.} = \frac{0.2605}{16.91} = 0.0154 \text{ in.}^2$$

$$A_{actual} = 2(1/16)(1/4) = 0.031 \text{ in.}^2$$

Use one strap $- A_{actual} = \frac{0.031}{2} = 0.0155 \text{ in.}^2$

Vertical Shear Check:

$$V = 1/64\ (10) = 0.15625 \text{ k}$$

$$A_{req.} = \frac{0.15625}{13.0} = 0.01202 \text{ in.}^2 \quad \text{SAFE}$$

$$M_R = \tfrac{1}{2}(1.375\ d^2 t) = \frac{1.375}{2}(1/4)^2(1/16) = 0.002686\ f_k$$

Comparison with strap connection for prototype. (See Section 4-3)

$n = 8$

	Prototype value divided by n or n^3.		Model value
d	$2 \div 8$	$= 1/4$	1/4
t	$\frac{1}{4} \div 8$	$= 1/32$	1/16
l	$\frac{1}{2} \div 8$	$= 1/16$	1/4
M_R	$1.375 \div (8)^3$	$= 0.002686$	0.002686

Critical load increments

M_R (prototype) = 1.375 fk

$\frac{1.375}{36.9} \times 1000$ ppf = 37.3 ppf (load on prototype to give moment of 1.375 fk at strap) (See Table V)

$\frac{37.3}{8}$ = 4.66 ppf (load on model)

$\frac{4.66}{2}$ = 2.33 pounds per wire. (load to stress strap to elastic limit)

4-5. Experimental Data: Articulated Model (See Table IX-a)

Semi-articulated Model (See Table IX-b)

Length of pointer arms for dials.

Point A - 9.82 in.

Point C - 9.86 in.

Point B - (articulated model) = $\frac{10.11 + 10.42}{2}$ = 10.265 in.

Point B - (semi-articulated model) = $\frac{10.11 + 10.51}{2}$ = 10.31 in

Sample computation (Point B - 8.8 pounds per wire)

$\frac{.029}{10.265}$ = .00283 radians

TABLE I

Moments of Inertia and Section Moduli of Segments

Segment	Aver. Depth (ins.)	Depth Minus .503 in.	I Flanges (in.^{4})	Depth Minus 1.006in.	I Web (in.^{4})	I Total (in.^{4})	S (in^3)
1	7.67	7.17	90.2	6.7	7.6	97.8	25.5
2	11.00	10.50	193.7	10.0	25.6	219.3	39.9
3	14.33	13.83	338	13.3	60.7	398.7	55.5
4	17.67	17.17	519	16.7	118.4	637.4	72.2
5	21.00	20.50	739	20.0	205.0	944	89.8
6	24.33	23.83	1001	23.3	325.0	1326	109.0
7	25.00	24.50	1057	24.0	354.0	1411	113.0
8	23.00	22.50	890	22.0	272.0	1162	101.0
9	21.00	20.50	739	20.0	205.0	944	89.8
10	19.00	18.50	602	18.0	149.0	761	79.1
11	17.00	16;50	479	16.0	105.0	584	68.6
12	15.00	14.50	370	14.0	70.2	440.2	58.6
13	13.00	12.50	274	12.0	44.2	318.2	49.0
14	11.00	10.50	193.7	10.0	25.6	219.3	39.9
15	9.00	8.50	127	8.0	13.1	140.1	31.2
16	7.00	6.50	74.4	6.0	5.5	79.9	22.8

TABLE I

TABLE II

Rotation and Vertical Deflection at Crown Hinge by Virtual Work

(Tension on outside of bent considered positive)

Segment	M	I	M/I	m_α	m_v	$\frac{M}{I} x m_\alpha$	$\frac{M}{I} x m_v$
1	+ 16.67	97.8	.170	+.042	+0.833	.0071	.1416
2	+ 50.0	219.3	.228	+.125	+2.500	.0285	.5700
3	+ 83.33	398.7	.209	+.209	+4.167	.0437	.8709
4	+116.67	637.4	.183	+.291	+5.833	.0533	1.0674
5	+150.0	944	.159	+.375	+7.500	.0596	1.1925
6	+183.33	1326	.1383	+.458	+9.167	.0633	1.2680
7	+180.5	1411	.1279	+.525	+9.5	.0671	1.2140
8	+144.5	1162	.124	+.575	+8.5	.0713	1.0540
9	+112.5	944	.119	+.625	+7.5	.0744	.8925
10	+ 84.5	751	.113	+.675	+6.5	.0763	.7345
11	+ 60.5	584	.104	+.725	+5.5	.0754	.5720
12	+ 40.5	440.2	.092	+.775	+4.5	.0713	.4140
13	+ 24.5	318.2	.077	+.825	+3.5	.0635	.2695
14	+ 12.5	219.3	.057	+.875	+2.5	.0499	.1425
15	+ 4.5	140.1	.032	+.925	+1.5	.0296	.0480
16	+ 0.5	79.9	.006	+.975	+0.5	.0059	.0030

(cont'd on next page)

TABLE II (cont'd)

Segment	M	I	M/I	m_α	m_v	$\frac{M}{I} \times m_\alpha$	$\frac{M}{I} \times m_v$
17	+ 0.5	79.9	.006	+.025	+0.5	.0002	.0030
18	+ 4.5	140.1	.032	+.075	+1.5	.0024	.0480
19	+ 12.5	219.3	.057	+.125	+2.5	.0071	.1425
20	+ 24.5	318.2	.077	+.175	+3.5	.0135	.2695
21	+ 40.5	440.2	.092	+.225	+4.5	.0207	.4140
22	+ 60.5	584	.104	+.275	+5.5	.0286	.5720
23	+ 84.5	751	.113	+.325	+6.5	.0367	.7345
24	+112.5	944	.119	+.375	+7.5	.0446	.8925
25	+144.5	1162	.124	+.425	+8.5	.0527	1.0540
26	+180.5	1411	.1279	+.475	+9.5	.0607	1.2140
27	+183.33	1326	.1383	+.458	+9.167	.0633	1.2680
28	+150.0	944	.159	+.375	+7.5	.0596	1.1925
29	+116.67	637.4	.183	+.291	+5.833	.0533	1.0674
30	+ 83.33	395.7	.209	+.209	+4.167	.0437	.8709
31	+ 50.0	219.3	.228	+.125	+2.5	.0285	.5700
32	+ 16.67	97.8	.170	+.042	+0.833	.0071	.1416
Total						1.3592	20.8086

$$\theta_B = \int_0^B \frac{M\, m_\alpha\, ds}{EI} = \frac{1.3592 \times 2 \times 144}{30{,}000} = .01304 \text{ Radians} = 44.8 \text{ minutes}$$

$$\Delta_B = \int_0^B \frac{M\, m_v\, ds}{EI} = \frac{20.8086 \times 2 \times 1728}{30{,}000} = 2.40 \text{ inches}$$

TABLE II

TABLE III

Rotation at Base Pins by Virtual Work

(Tension on outside of bent considered positive)

Section	M	m
A D	$16.67\,x$	$1 - x/24$
D B	$200-20\,x + x^2/2$	$0.5 - x/40$
F B	$200-20\,x + x^2/2$	$-0.5 + x/40$
C F	$16.67x$	$- x/24$

Segment	M	m	I	$\frac{Mm}{I}$
1	+ 16.67	+.958	97.8	+.1632
2	+ 50.00	+.875	219.3	+.1993
3	+ 83.33	+.79}	398.7	+.1652
4	+116.67	+.709	637.4	+.1297
5	+150.00	+.625	944	+.0994
6	+183.33	+.542	1326	+.0750
7	+180.50	+.475	1411	+.0607
8	+144.50	+.425	1162	+.0528
9	+112.50	+.375	944	+.0446
10	+ 84.50	+.325	751	+.0366
11	+ 60.50	+.275	584	+.0285
12	+ 40.50	+.225	440.2	+.0207
13	+ 24.50	+.175	318.2	+.0135
14	+ 12.50	+.125	219.3	+.0071
15	+ 4.50	+.075	140.1	+.0024
16	+ 0.50	+.025	79.9	+.0016

(cont'd on next page)

TABLE III (cont'd)

Segment	M	m	I	$\frac{Mm}{I}$
17	+ 0.50	-.025	79.9	-.0016
18	+ 4.50	-.075	140.1	-.0024
19	+ 12.50	-.125	219.3	-.0071
20	+ 24.50	-.175	318.2	-.0135
21	+ 40.50	-.225	440.2	-.0207
22	+ 60.50	-.275	584	-.0285
23	+ 84.50	-.325	751	-.0366
24	+112.50	-.375	944	-.0446
25	+144.50	-.425	1162	-.0528
26	+180.50	-.475	1141	-.0607
27	+183.33	-.458	1326	-.0634
28	+150.00	-.375	944	-.0596
29	+116.67	-.291	637.4	-.0533
30	+ 83.33	-.209	398.7	-.0437
31	+ 50.00	-.125	219.3	-.0285
32	+ 16.67	-.042	97.8	-.0072
Total				+.5761

$$\theta_A = \sum \frac{M\,m_\alpha\,ds}{EI} = \frac{.5761 \times 2 \times 144}{30{,}000} = .0055306 \text{ Radians}$$

$$= 19.01 \text{ Minutes}$$

TABLE III

TABLE IV

Rotation at Crown and Base Hinges

By Amirikian Method

Segment	M	I	M/I	Y	$\frac{My}{I}$
1	16.67	97.8	.170	1	0.170
2	50.00	219.3	.228	3	0.684
3	83.33	398.7	.209	5	1.045
4	116.67	637.4	.183	7	1.281
5	150.00	944	.159	9	1.431
6	183.33	1326	.138	11	1.521
7	180.50	1411	.128	12	1.536
8	144.50	1162	.124	12	1.488
9	112.50	944	.119	12	1.428
10	84.50	751	.113	12	1.356
11	60.50	584	.104	12	1.248
12	40.50	440.2	.092	12	1.104
13	24.50	318.2	.077	12	0.924
14	12.50	219.3	.057	12	0.684
15	4.50	140.1	.032	12	0.384
16	0.50	72.9	.006	12	0.072
			1.939		16.355

ds = 2.0

$\sum \frac{Myds}{I} = 32.71$ $\sum \frac{Mds}{I} = 3.878$

$\theta_{BA} = \frac{32.71 \times 144}{12 \times 30,000} = 0.013084$ radians = 44.8 minutes

$\theta_{AB} = \left(\frac{3.878 \times 144}{30,000}\right) - .013084 = .0186144 - .013084$

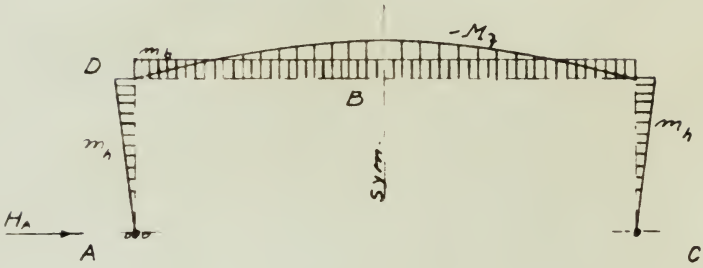
$= .0055304$ radians = 19.01 minut

TABLE IV

TABLE V

Deflections by Virtual Work

Assuming Rigid Joint at B



Segment	M	m_h	I	M/I	m_h^2/I
1	---	+ 1	97.8	---	+0.0102
2	---	+ 3	219.3	---	+0.0410
3	---	+ 5	398.7	---	+0.0627
4	---	+ 7	637.4	---	+0.0769
5	---	+ 9	944.0	---	+0.0858
6	---	+11	1326.0	---	+0.1077
7	- 19.5	+12	1411.0	-0.0138	+0.1021
8	- 55.5	+12	1162.0	-0.0477	+0.1238
9	- 87.5	+12	944.0	-0.0928	+0.1527
10	-115.5	+12	751.0	-0.1540	+0.1921
11	-139.5	+12	584.0	-0.2389	+0.2465
12	-159.5	+12	440.2	-0.3622	+0.3271
13	-175.5	+12	318.2	-0.5520	+0.4525
14	-187.5	+12	219.3	-0.8560	+0.6570
15	-195.5	+12	140.1	-1.3950	+1.0280
16	-199.5	+12	79.9	-2.4880	+1.8020
				-6.2004	+5.4681

$$\sum \frac{M m_h \Delta_x}{I} = E\Delta_{H_A} = -6.2004 \times 2 \times 2 \times 12 = -297.62$$

$$\sum \frac{m_h^2 \Delta_x}{I} = E\delta_{H_A} = +5.4681 \times 2 \times 2 = +21.87$$

$$H_A = \frac{297.62}{21.87} = 13.60^{k}$$

$$M_D = 13.6 \times 12 = 163.1 \text{ fk}$$

$$M_B = 200.0 - 163.1 = 36.9 \text{ fk}$$

TABLE V

TABLE VI

Moments of Inertia of Model

$$\left(I_M = \frac{I_p}{(8)^4}\right)$$

Segment	I_p	I_M
A	56.5	.01379
1	97.8	.02388
2	219.3	.05350
3	398.7	.09740
4	637.4	.15561
5	944.0	.23040
6	1324.4	.32333
D	1544.5	.37707
7	1410.4	.34433
8	1162.0	.28400
9	944.0	.23040
10	751.0	.18320
11	584.0	.14250
12	440.2	.10750
13	318.2	.07780
14	219.3	.05350
15	140.1	.03422
16	79.9	.01951
B	56.5	.01379

TABLE VI

TABLE VII

Computation of Model Dimensions

(1) Seg-ment	(2) t_{web}	(3) $=d_{dp/8}$	(4) t_{fl}*	(5) $d-2t_{fl}$	(6) I_{web}	(7) Total $I_{req.}$
A	.130	0.750	.0613	.627	.00267	.01379
1	.130	0.959	.0613	.836	.00636	.02385
2	.130	1.375	.0613	1.252	.02127	.05350
3	.130	1.791	.0613	1.668	.05019	.09740
4	.130	2.209	.0613	2.086	.09849	.15561
5	.130	2.625	.0613	2.502	.16970	.23040
6	.130	3.041	.0613	2.918	.26894	.32333
D	.130	3.250	.0613	3.127	.33129	.37707
7	.130	3.125	.0613	3.002	.29313	.34432
8	.130	2.875	.0613	2.752	.22683	.28400
9	.130	2.625	.0613	2.502	.16970	.23040
10	.130	2.375	.0613	2.252	.12375	.18320
11	.130	2.125	.0613	2.002	.08695	.14250
12	.130	1.875	.0613	1.752	.05827	.10750
13	.130	1.625	.0613	1.502	.03672	.07780
14	.130	1.375	.0613	1.252	.02127	.05360
15	.130	1.125	.0613	1.002	.01090	.03422
16	.130	0.875	.0613	0.752	.00461	.01951

* t_{fl} - assumed thickness of flange plates

(Cont'd on next page)

TABLE VII (cont'd)

(1) Segment	(8) $I_{req.}$ (flange)	(9) Col(6)+ t'_{f2}**	(10) $(\text{Col.}9)^2$	(11) $\frac{2x.068\,(\text{Col.}10)}{4}$	(12) flange width = (Col. 8) ÷ (Col.11)
A	.01112	0.695	0.48303	.01642	.677
1	.01749	0.904	0.81902	.02785	.628
2	.03223	1.320	1.74240	.05924	.544
3	.04721	1.735	3.01022	.10235	.461
4	.05712	2.155	4.64402	.15790	.362
5	.06070	2.570	6.60490	.22457	.270
6	.05439	2.985	8.91022	.30295	.180
D	.04578	3.195	10.20802	.34707	.132
7	.05120	3.070	9.42490	.32045	.160
8	.05817	2.820	7.95240	.27038	.215
9	.06070	2.570	6.60490	.22457	.270
10	.05944	2.320	5.38240	.18300	.325
11	.05555	2.070	4.28490	.14569	.381
12	.04923	1.820	3.31240	.11262	.437
13	.04108	1.570	2.46490	.08381	.490
14	.03223	1.320	1.74240	.05924	.546
15	.02332	1.070	1.14490	.03893	.599
16	.01490	0.820	0.67240	.02286	.652

** t'_{f1} - actual thickness of flange plates = .068"

TABLE VII

TABLE VIII

Alternate Model Design

Let $n = 20$	$L/L_2 = n$	$I/I_2 = n^4 m$
$m = 2$	$W/W_2 = n^2 m$	$\theta/\theta_2 = 1$
$E_2 = E$		

$t = 0.1300$ inches

Segment*	I_p	I_2	$\frac{12I_2}{t}$	$d(in)=(\frac{12I_2}{t})^{\frac{1}{3}}$
A	56.5	.000177	.01634	0.254
1	97.8	.000306	.02825	0.305
2	219.3	.000685	.06323	0.396
3	398.7	.001246	.11502	0.486
4	637.4	.001992	.18388	0.568
5	944.0	.002950	.27231	0.648
D"	1303.5	.004073	.37597	0.722
D'	1393.0	.004353	.40181	0.738
8	1162.0	.003631	.33517	0.695
9	944.0	.002950	.27231	0.648
10	751.0	.002347	.21664	0.600
11	584.0	.001825	.16846	0.551
12	440.2	.001376	.12702	0.503
13	318.2	.000994	.09175	0.451
14	219.3	.000685	.06323	0.396
15	140.1	.000438	.04043	0.344
16	79.9	.000250	.02307	0;285
B	56.5	.000177	.01634	0.254

*Segment designations are the same as those used in the first model as shown in Fig. 4 and Fig. 5.

TABLE VIII

TABLE IX

Model Loading and Deflection Data

IX-a

Articulated Model

Load		Point A			Point B		
Per Wire	Per Foot	Dial Reading	Diff.	Angle	Dial Reading	Diff.	Angle
0	0	300			600		
8.8	17.6	317	-17		571	29	.00283
30.0	60.0	298	2		519	81	.00789
45.0	90.0	286	14		481	119	.01160
--	118.0	274	26		450	150	.01461

Load		Point B(Vert.)		Point C		
Per Wire	Per Foot	Dial Reading	Diff.	Dial Reading	Diff.	Angle
0	0	400		300		
8.8	17.6	459	59	293.5	6.5	.00066
30.0	60.0	566	166	277	23	.00234
45.0	90.0	647	247	265	35	.00356
--	118.0	713	313	252	48	.00487

Dial Readings in 0.001 inch.

Angles in radians.

IX-a

TABLE IX

Model Loading and Deflection Data

IX-b

Semi-Articulated Model

Load		Point A			Point B		
Per Wire	Per Foot	Dial Reading	Diff.	Angle	Dial Reading	Diff.	Angle
0	0	730	--		420	--	
8.8	17.6	677	53		390	30	.00290
30.0	60.0	651	79		336	84	.00814
45.0	90.0	630	100		289	131	.01269
62.5	125.0	607	127		223	197	.01909
0	0	725	--		340	--	
62.5	125.0	659	66		191	149	.01452

Load		Point B(Vert.)		Point C		
Per Wire	Per Foot	Dial Reading	Diff.	Dial Reading	Diff.	Angle
0	0	490	--	920	--	
8.8	17.6	551	61	914	6	.00061
30.0	60.0	675	185	894	26	.00264
45.0	90.0	770	280	880	40	.00400
62.5	125.0	890	400	864	56	.00569
0	0	536	--	919	--	
62.5	125.0	893	357	864	55	.00558

Dial Readings in 0.001 inch.

Angles in radians.

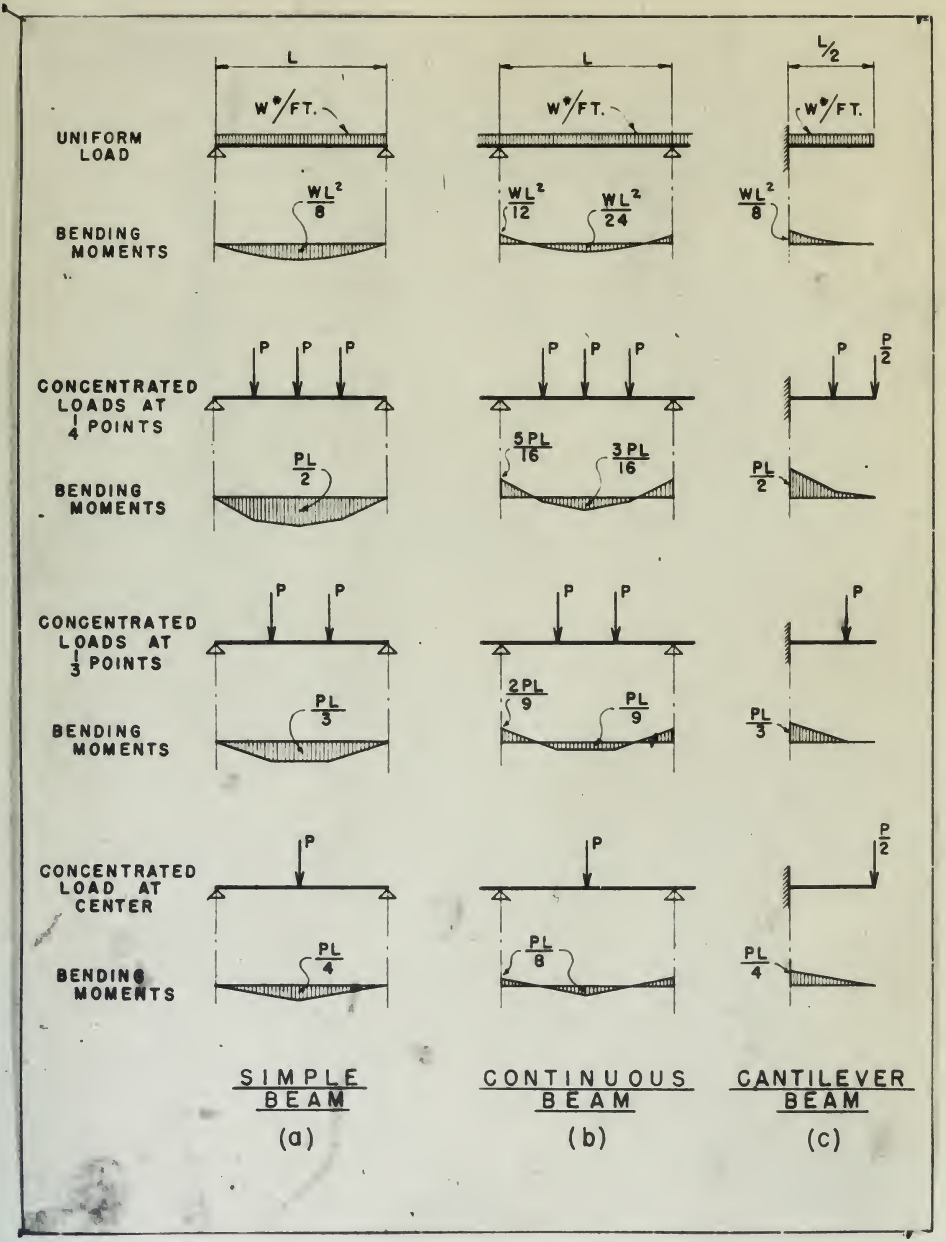
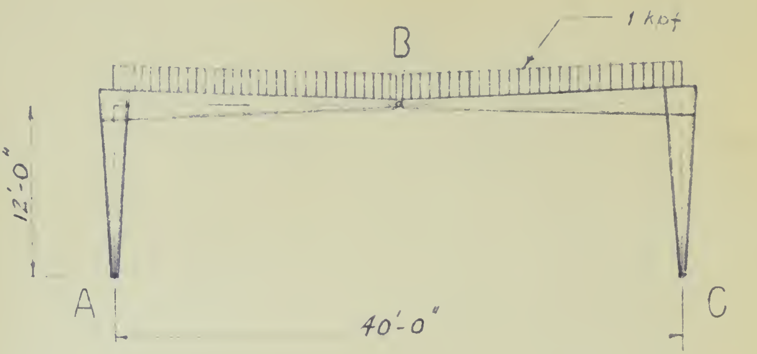
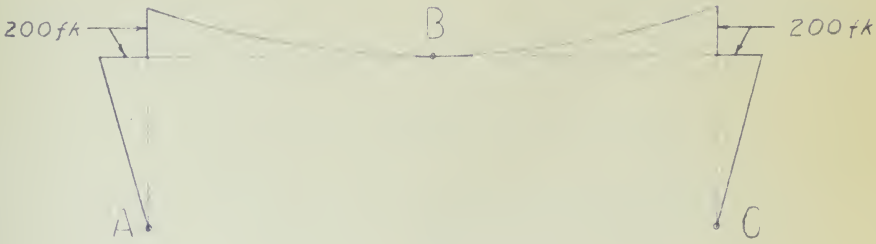

L
W#/FT.
UNIFORM LOAD
$\frac{WL^2}{8}$
BENDING MOMENTS
L
W#/FT.
$\frac{WL^2}{12}$
$\frac{WL^2}{24}$
L/2
W#/FT.
$\frac{WL^2}{8}$
CONCENTRATED LOADS AT $\frac{1}{4}$ POINTS
P
P
P
$\frac{PL}{2}$
BENDING MOMENTS
P
P
P
$\frac{5PL}{16}$
$\frac{3PL}{16}$
P
$\frac{P}{2}$
$\frac{PL}{2}$
CONCENTRATED LOADS AT $\frac{1}{3}$ POINTS
P
P
$\frac{PL}{3}$
BENDING MOMENTS
P
P
$\frac{2PL}{9}$
$\frac{PL}{9}$
P
$\frac{PL}{3}$
CONCENTRATED LOAD AT CENTER
P
$\frac{PL}{4}$
BENDING MOMENTS
P
$\frac{PL}{8}$
$\frac{P}{2}$
$\frac{PL}{4}$
SIMPLE BEAM
(a)
CONTINUOUS BEAM
(b)
CANTILEVER BEAM
(c)

FIG. 1

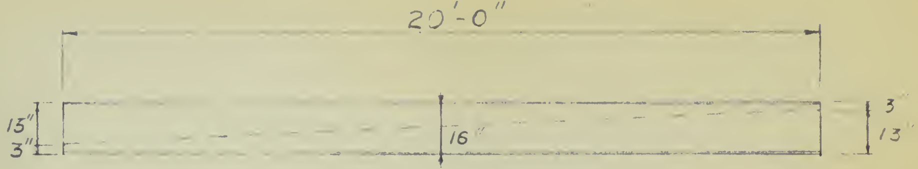


PROTOTYPE – DIMENSIONS AND LOADING

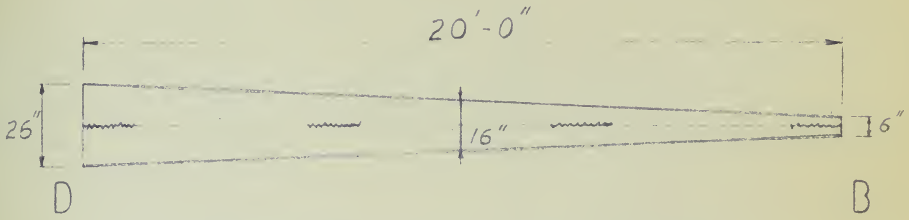


MOMENT DIAGRAM – TENSION SIDE

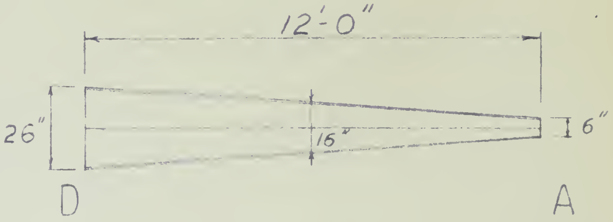
FIG. 2



16WF40 - PROPOSED CUT



FINISHED BEAM SECTION



FINISHED COLUMN SECTION

FIG. 3

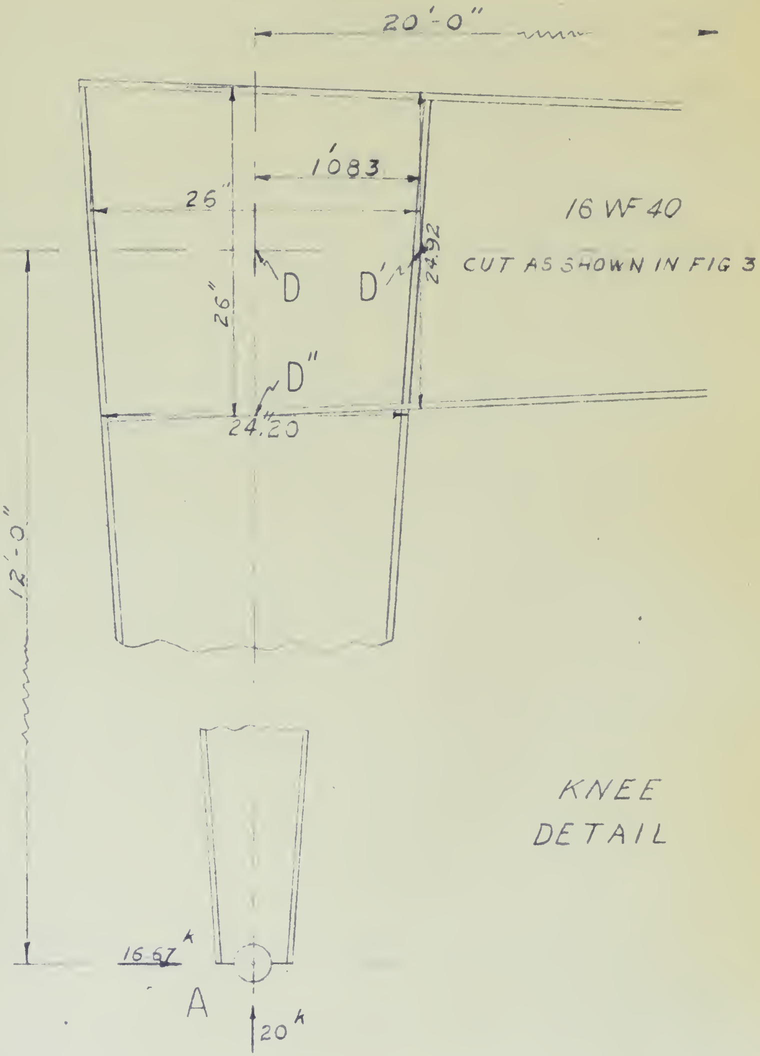
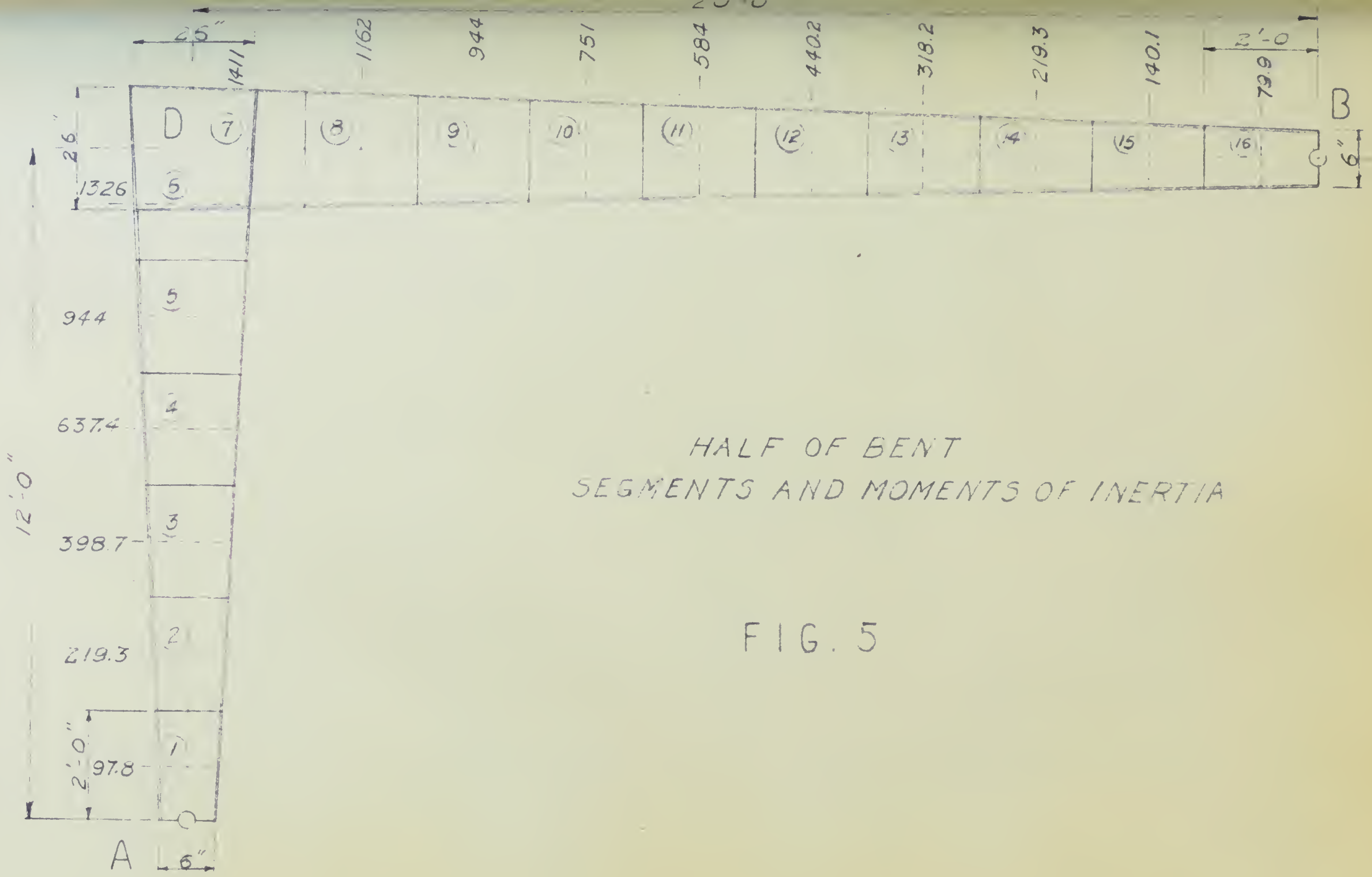

20'-0"
1'083
26"
16 WF 40
CUT AS SHOWN IN FIG 3
24.92
D
D'
26"
D"
24.20
12'-0"
KNEE
DETAIL
16.67 k
A
20 k

FIG. 4



HALF OF BENT
SEGMENTS AND MOMENTS OF INERTIA

FIG. 5

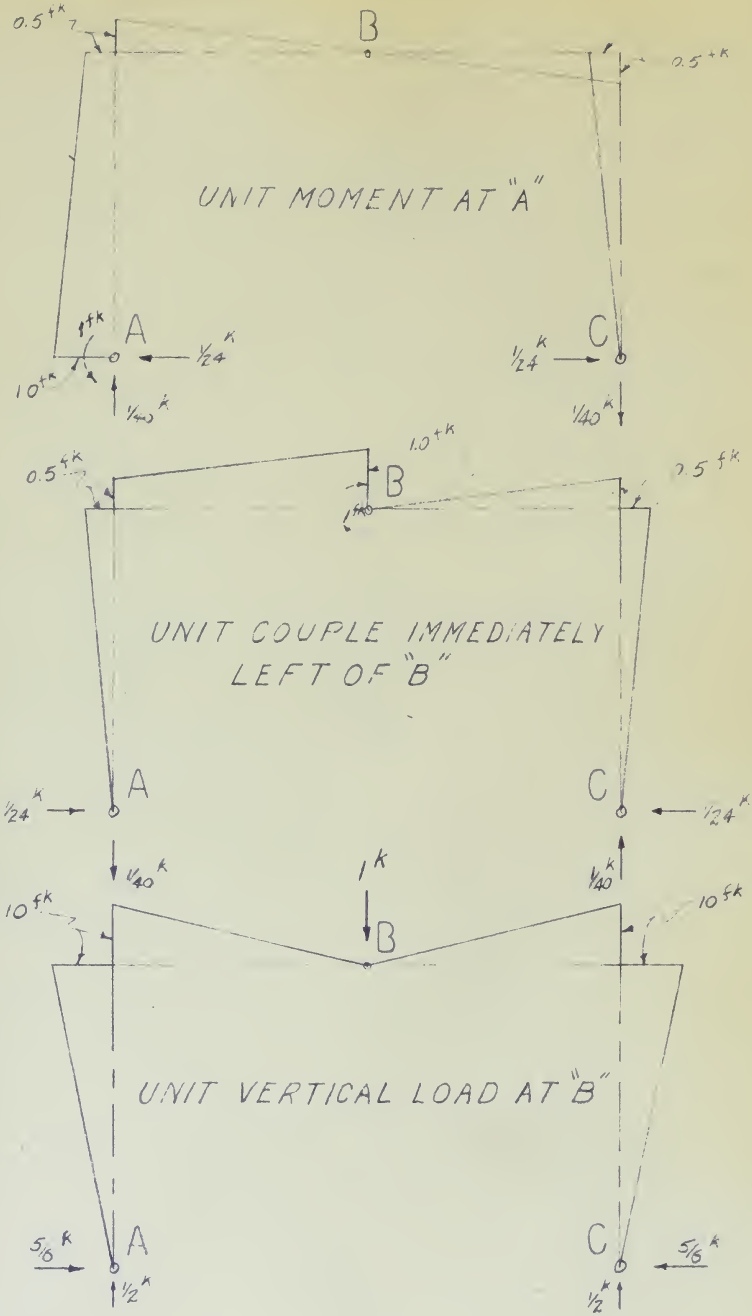
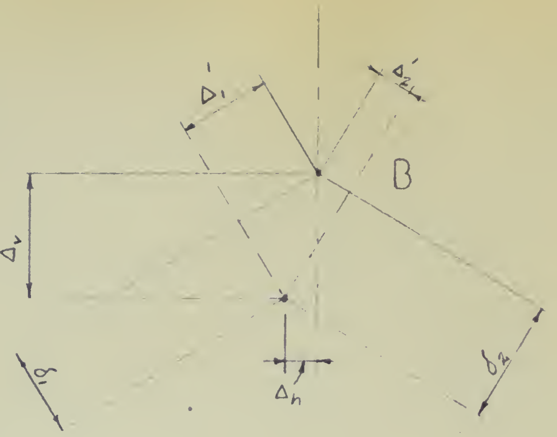

0.5 ʼK
B
0.5 ʼK
UNIT MOMENT AT "A"
1ʼK
A
1/24 K
10ʼK
1/40 K
1/24 K
C
1/40 K
1.0ʼK
0.5ʼK
B
0.5ʼK
UNIT COUPLE IMMEDIATELY LEFT OF "B"
A
1/24 K
C
1/24 K
1/40 K
1 K
1/40 K
10ʼK
B
10ʼK
UNIT VERTICAL LOAD AT "B"
A
5/6 K
1/2 K
C
5/6 K
1/2 K

FIG. 6



DETAIL B

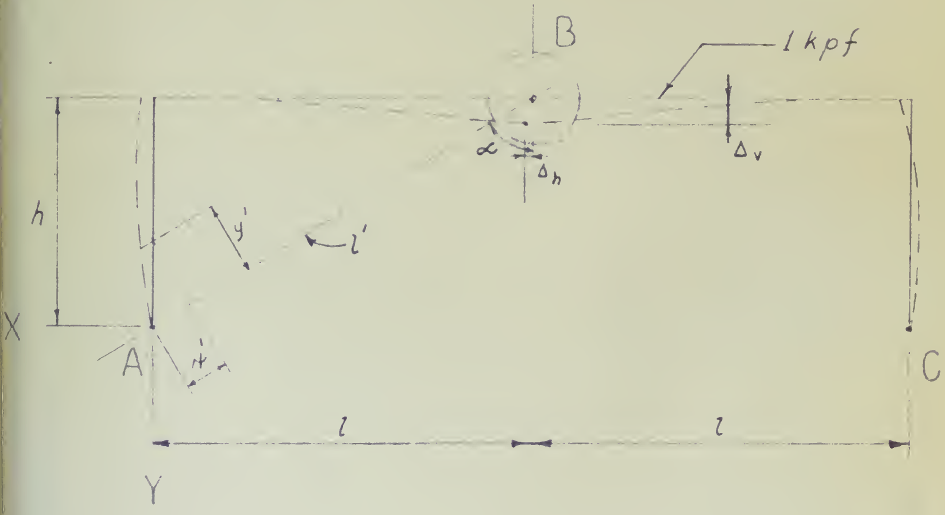
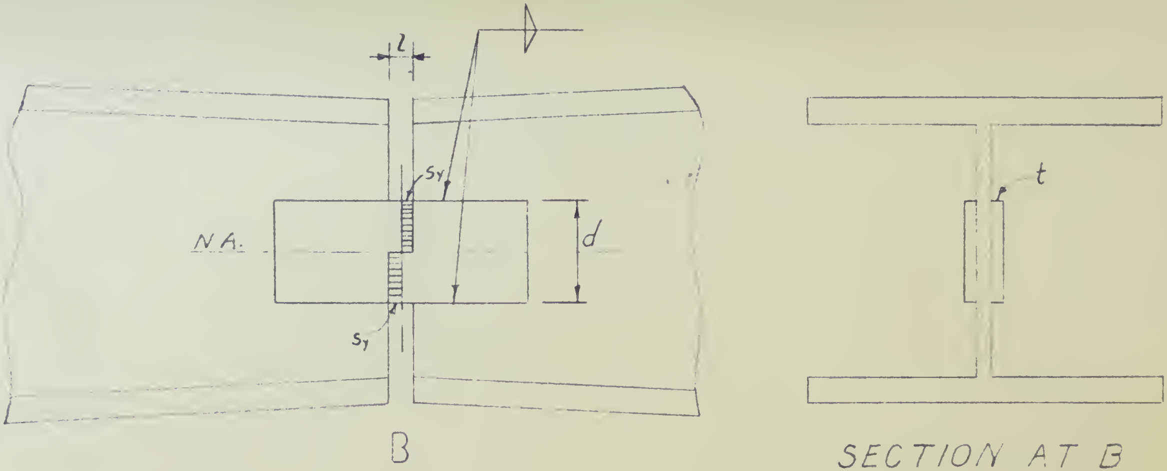


ILLUSTRATION OF AMIRIKIAN FORMULA DERIVATION

FIG. 7

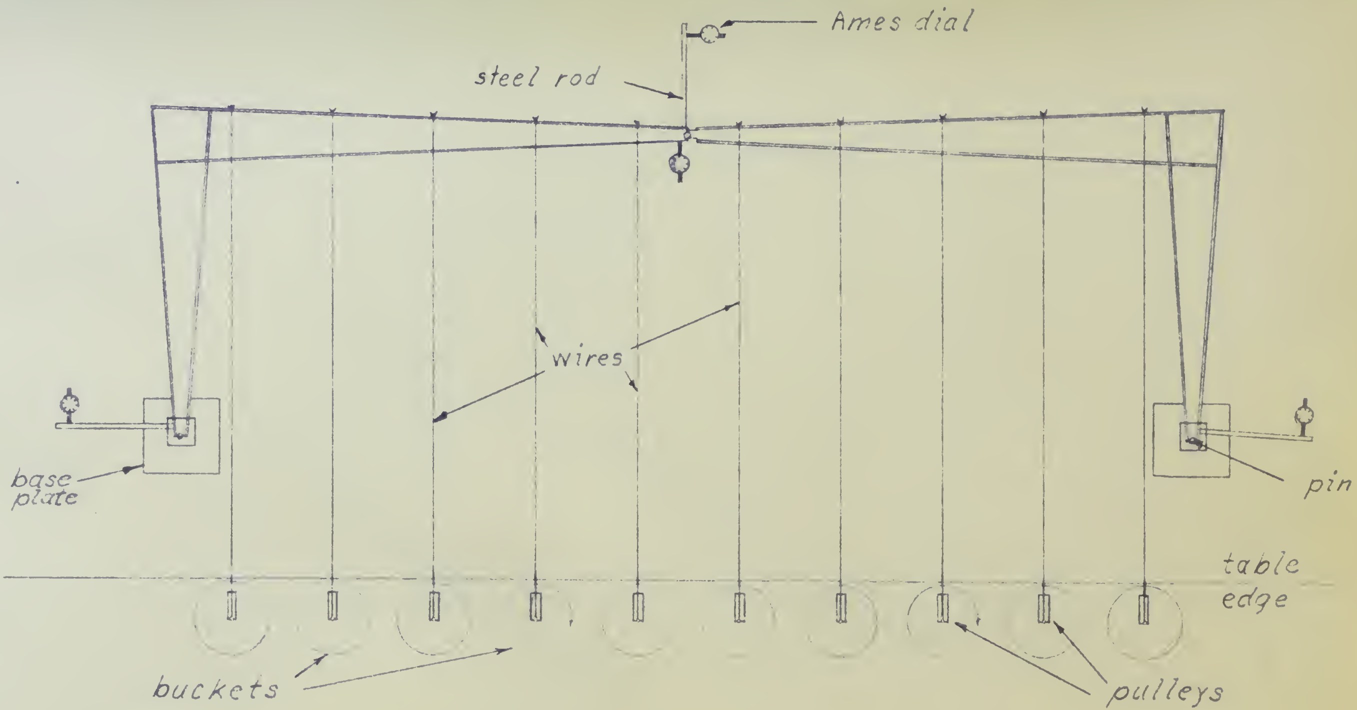


SECTION AT B

SEMI-FLEXIBLE CONNECTION FOR PROTOTYPE.

DETAILS AND STRESS DISTRIBUTION

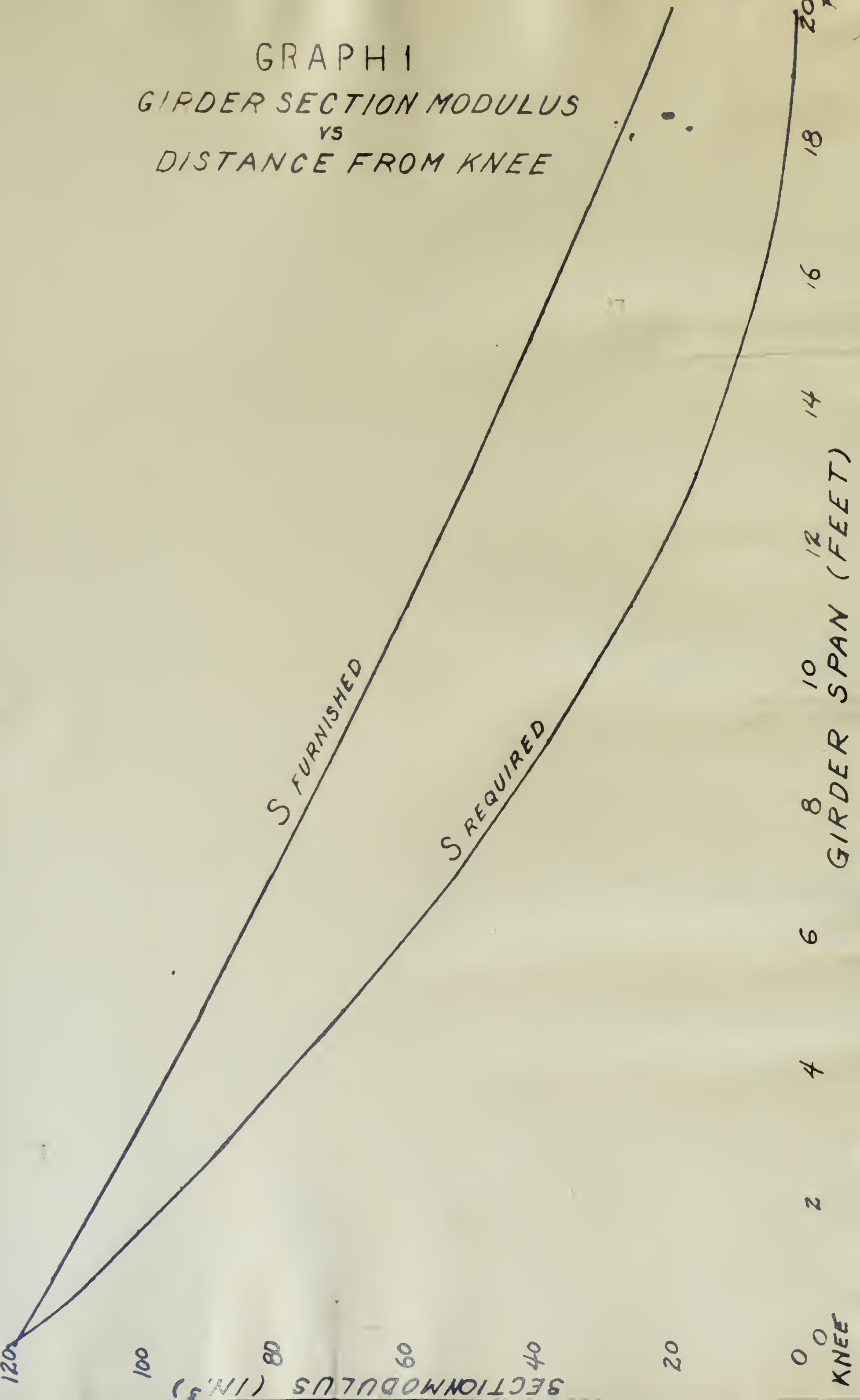
FIG. 8



MOUNTING OF MODEL

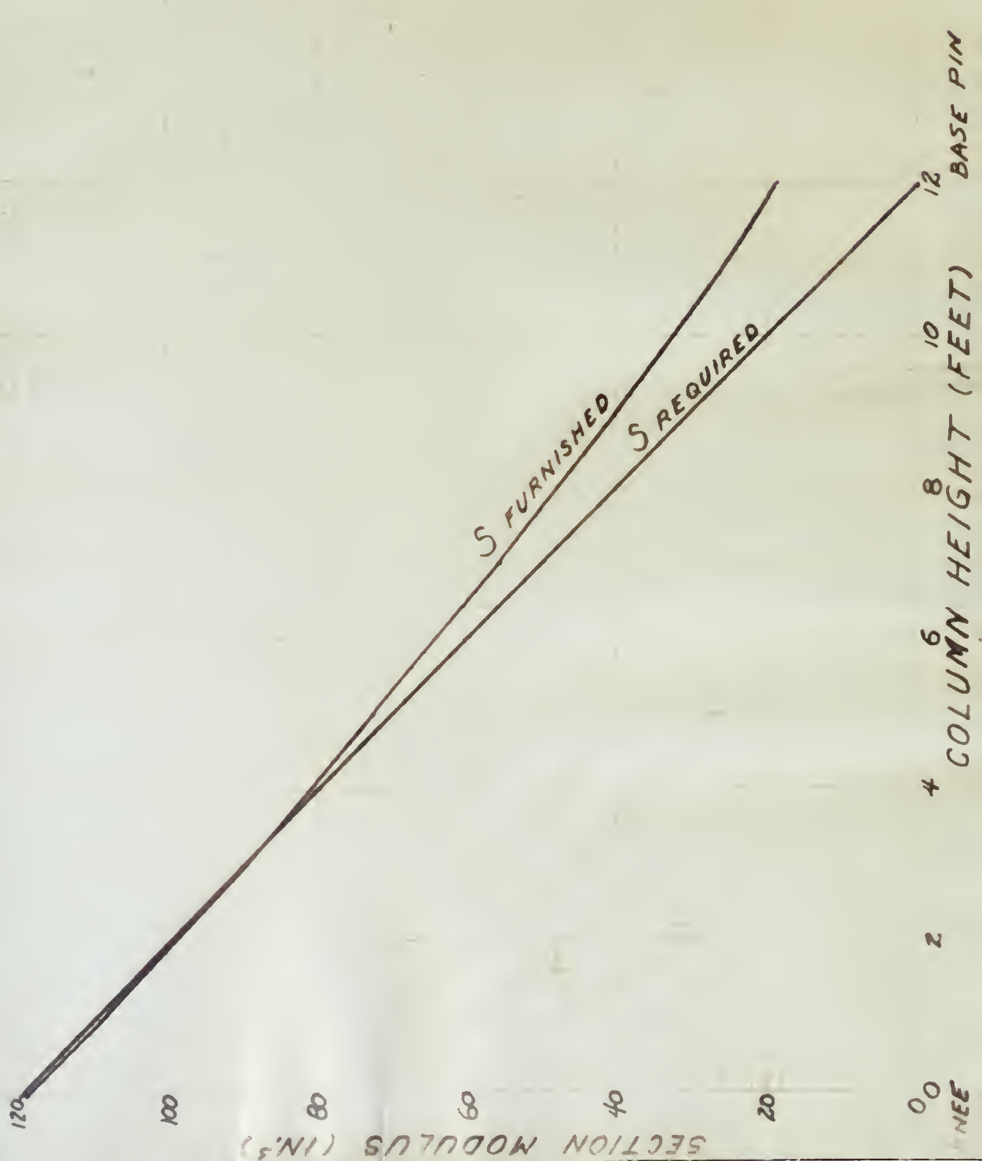
FIG. 9

GRAPH 1
GIRDER SECTION MODULUS
VS
DISTANCE FROM KNEE



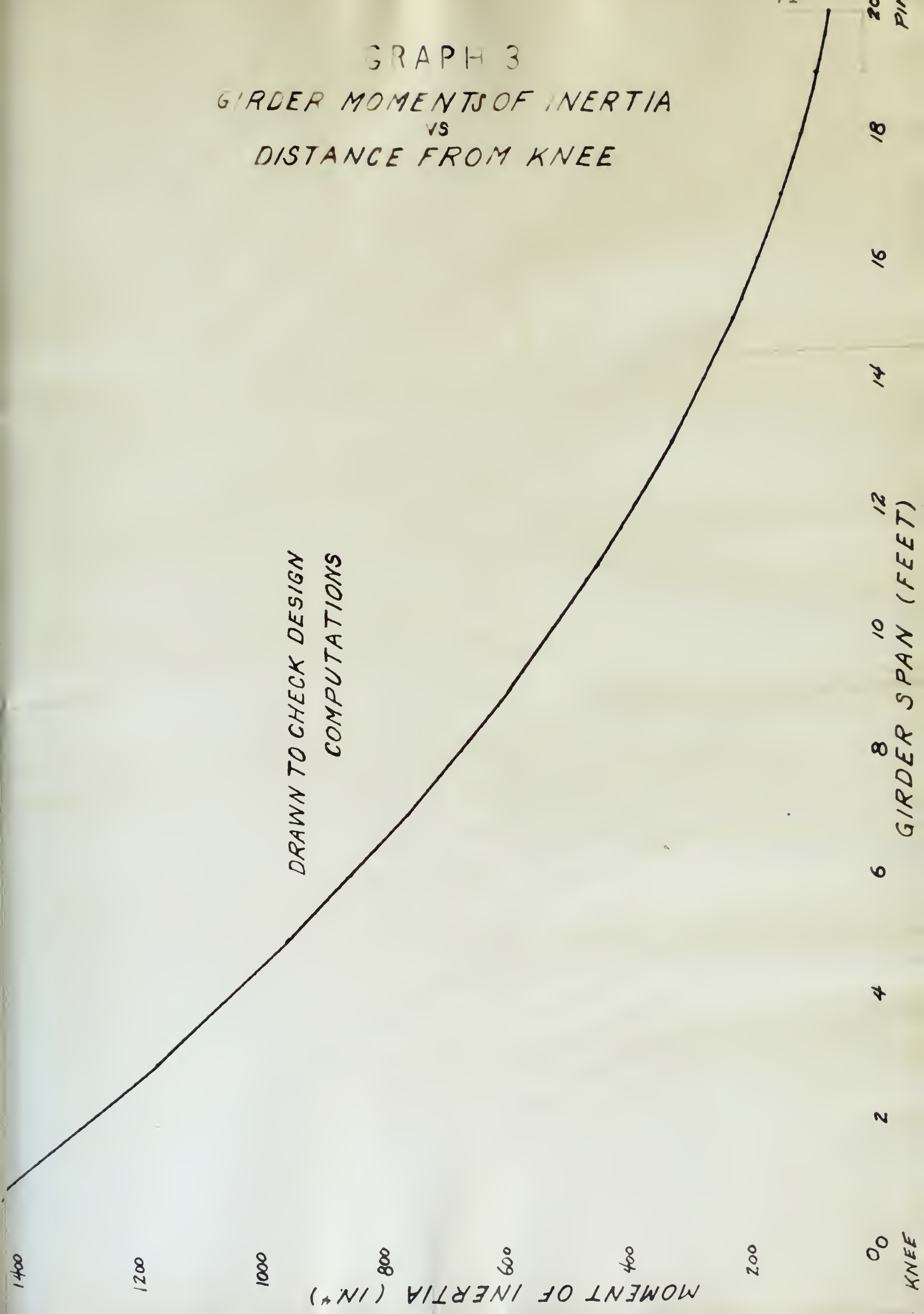
GRAPH 2

COLUMN SECTION MODULUS
VS
DISTANCE FROM KNEE



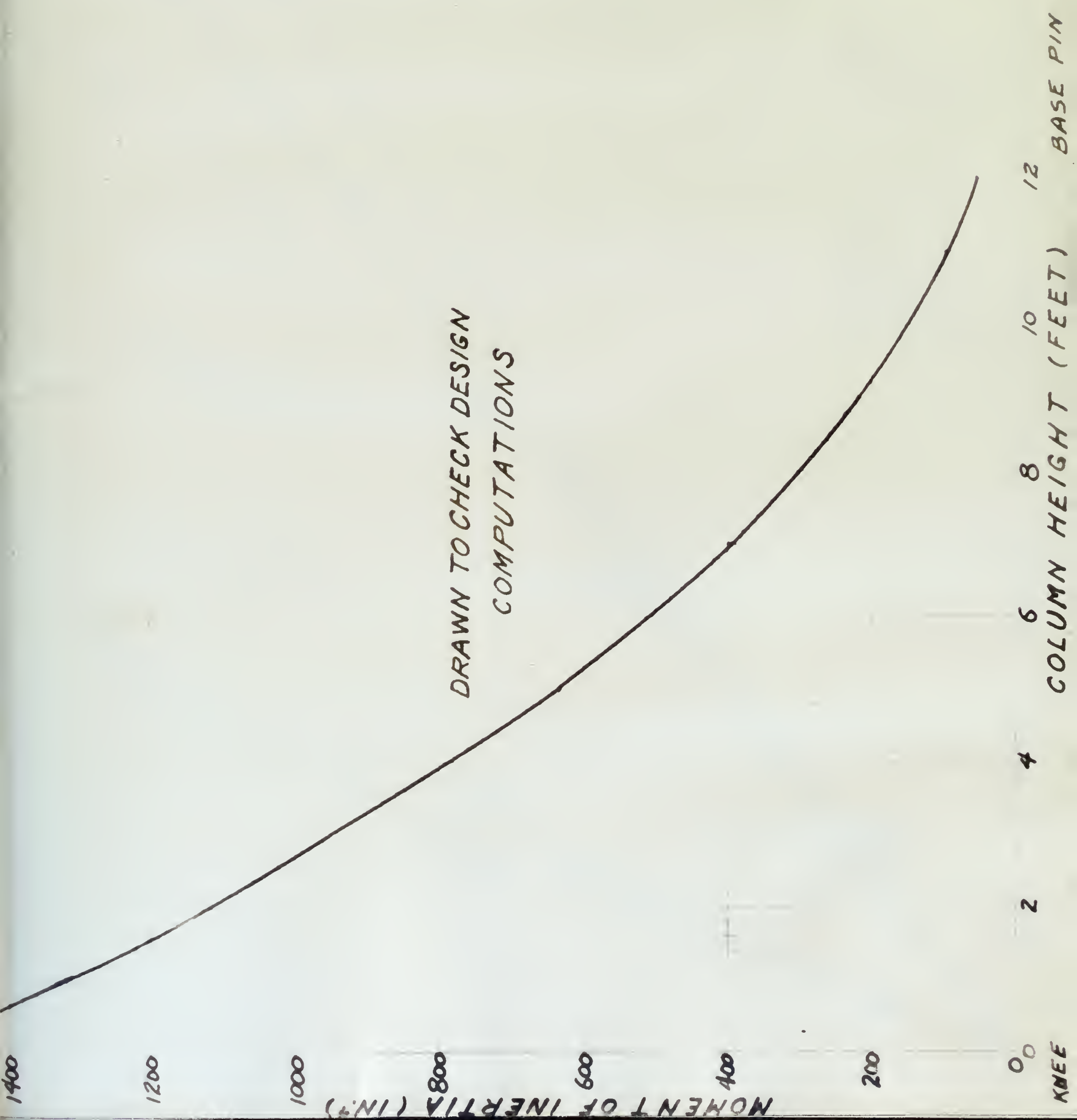
GRAPH 3

GIRDER MOMENTS OF INERTIA VS DISTANCE FROM KNEE



GRAPH 4

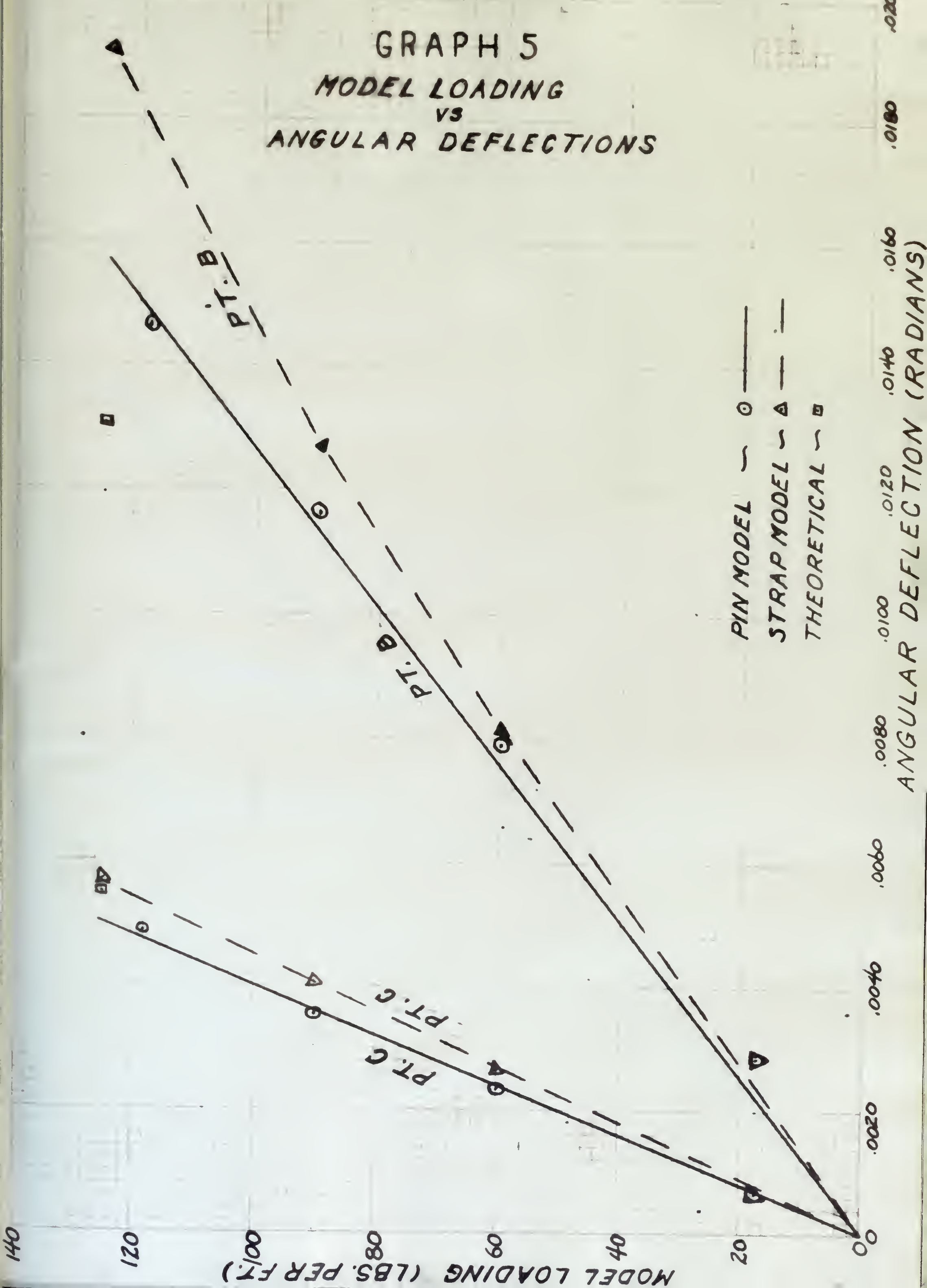
COLUMN MOMENTS OF INERTIA VS DISTANCE FROM KNEE



DRAWN TO CHECK DESIGN COMPUTATIONS

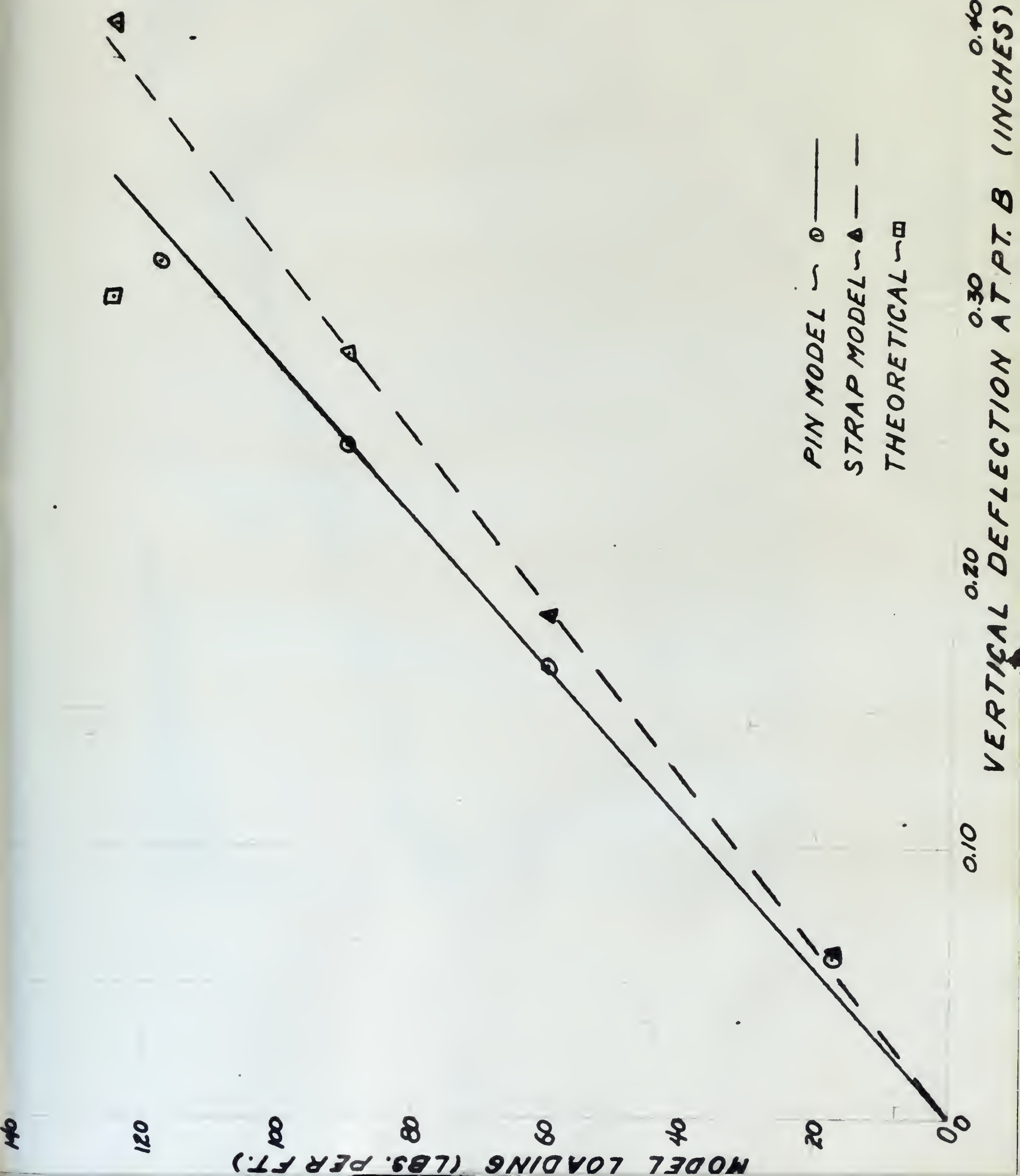
GRAPH 5

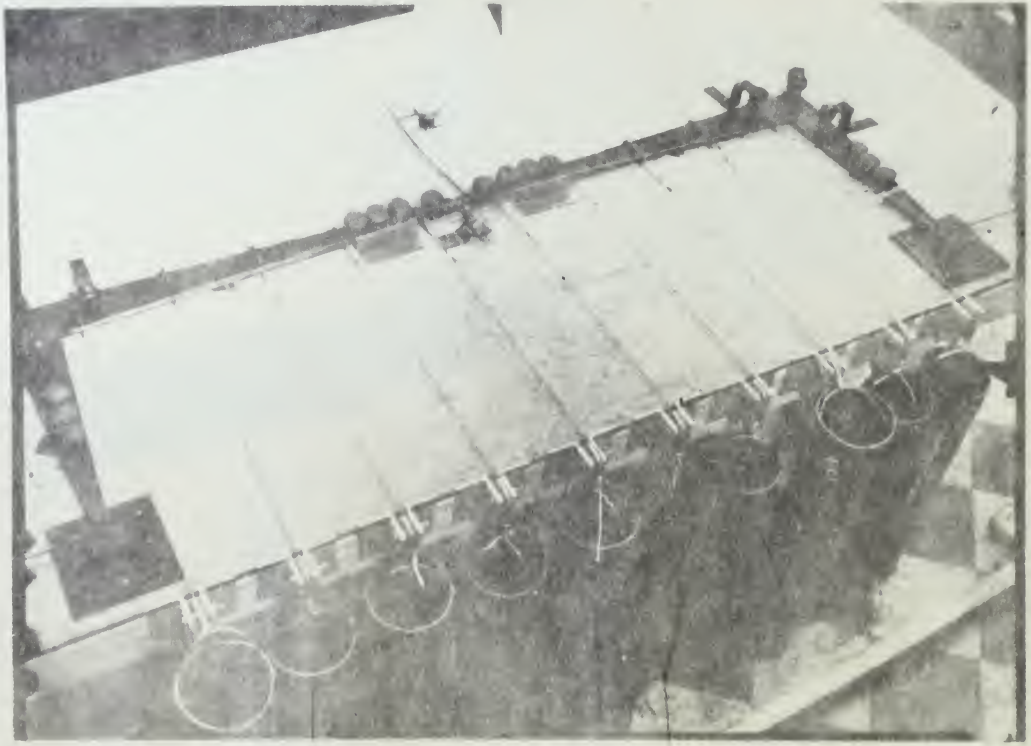
MODEL LOADING VS ANGULAR DEFLECTIONS



GRAPH 6

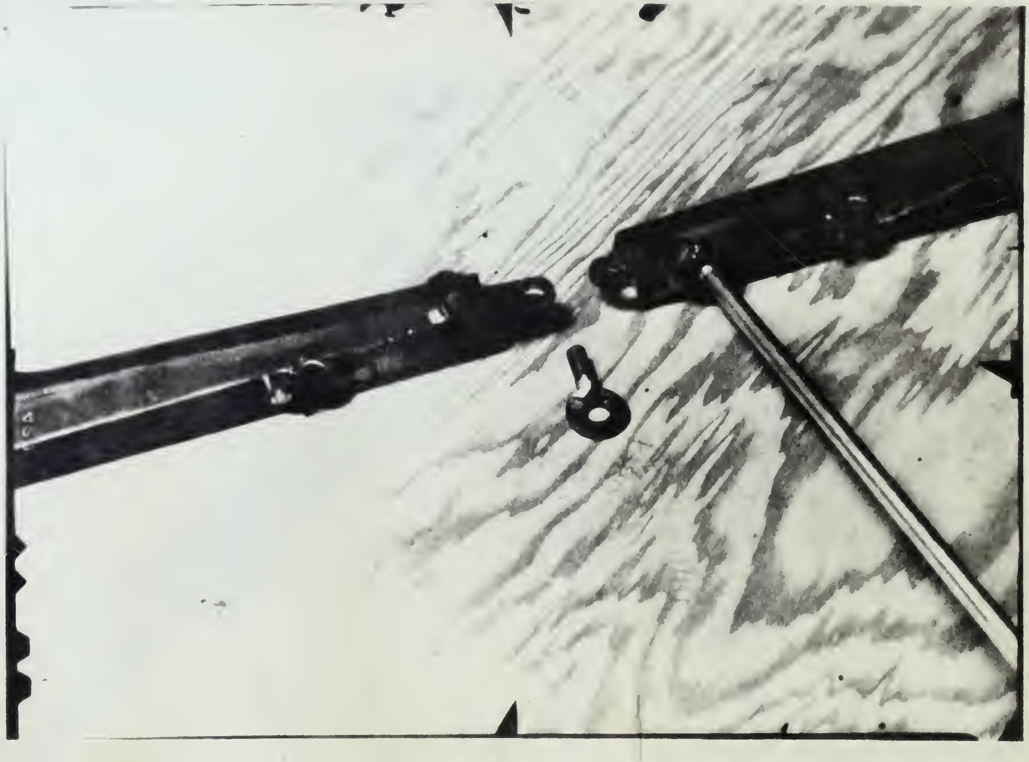
MODEL LOADING
VS
VERTICAL DEFLECTION
AT POINT B





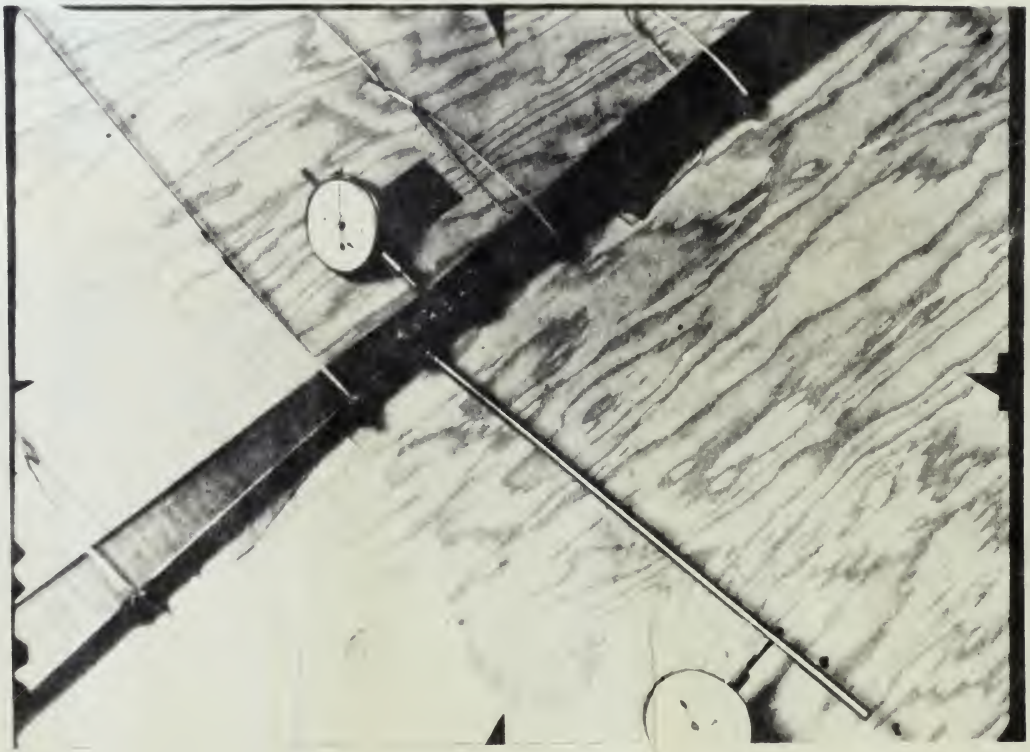
Photograph 1

Overall Picture of Model Mounting



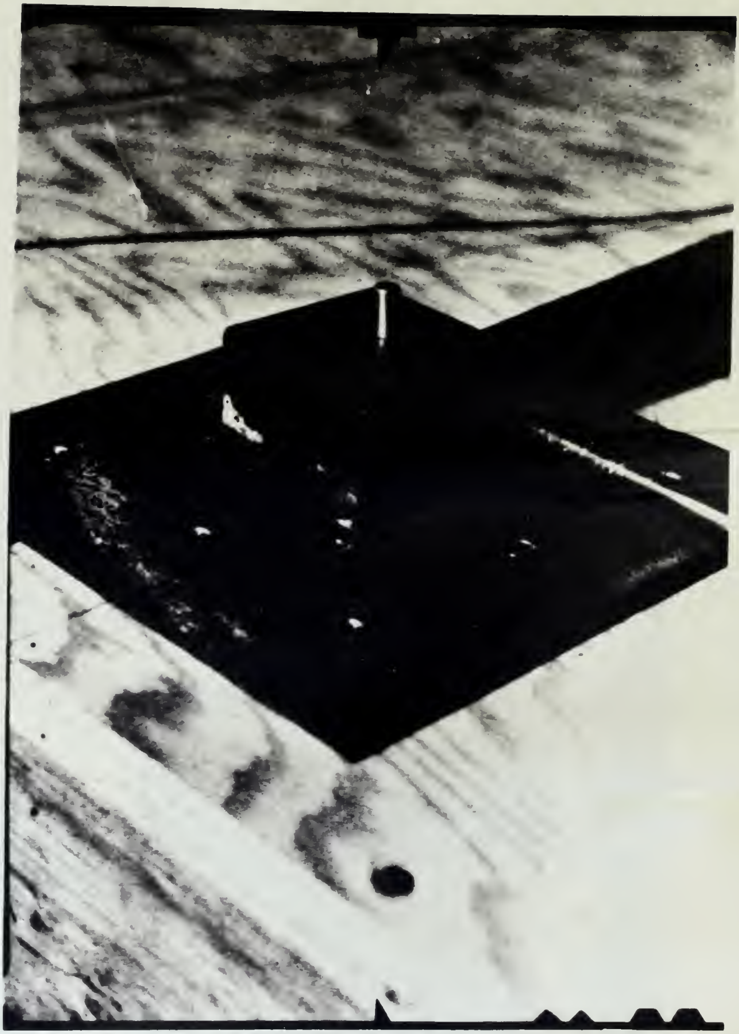
Photograph 2

Pin Connection at Crown of
Articulated Model



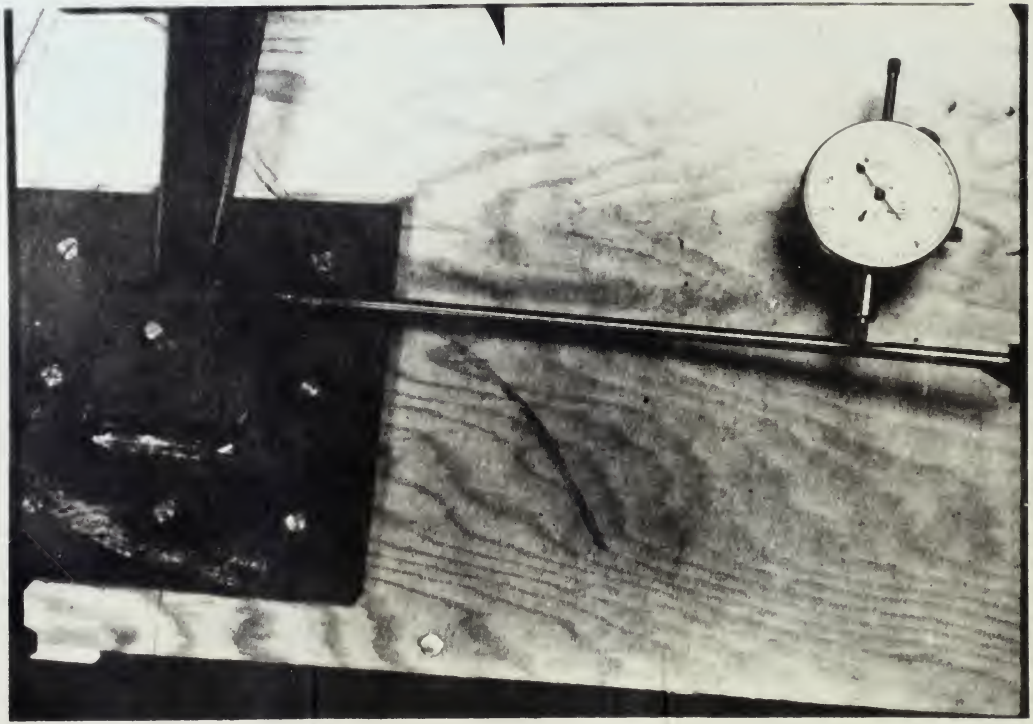
Photograph 3

Welded Strap Connection at Crown
of Semi-Articulated Model



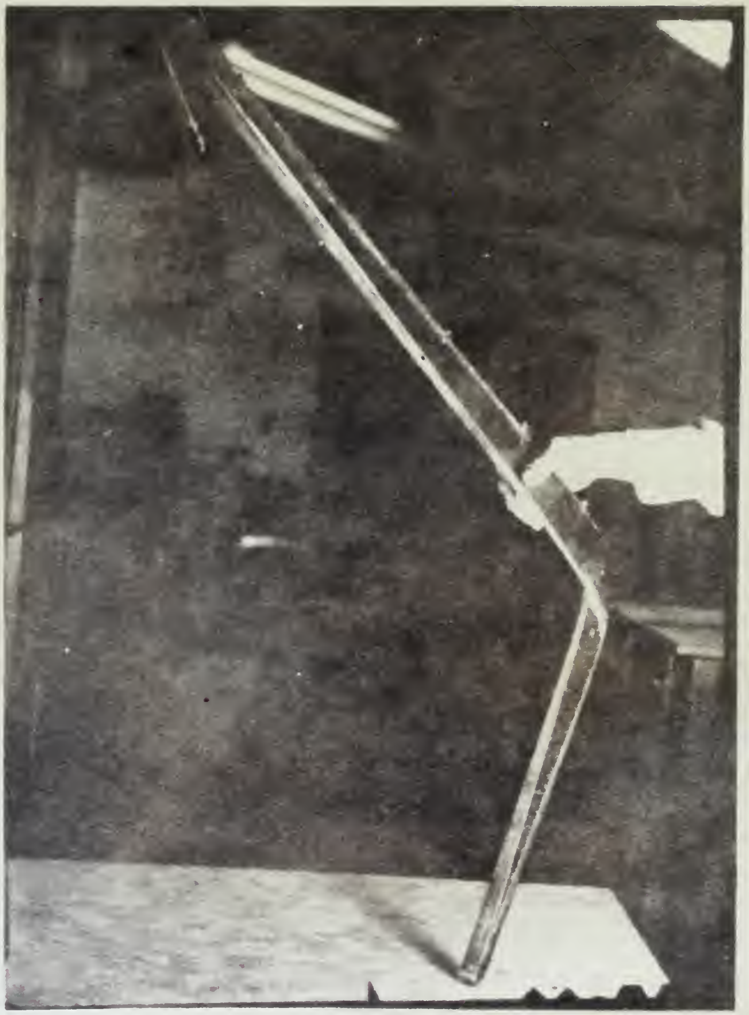
Photograph 4

Base Pin Detail



Photograph 5

Instrumentation at Base Pin



Photograph 7

Buckled Half Frame

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